STATEMENT OF TEACHING PHILOSOPHY, EXPERIENCES, AND INTERESTS

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As a postdoc at UC–Davis, I have taught three quarters of calculus and twice taught an upper division course in convex geometry, as well as a graduate course in algebraic combinatorics. Over this past summer, I supervised four students in research in algebraic combinatorics and in graph theory. This quarter, I am teaching a course on differential equations. While a graduate student at UC–Berkeley, I also served as a teaching assistant for calculus, linear algebra, and an introductory computer science course. In addition I had the benefit of learning from excellent teachers while an undergraduate at Williams College.

I believe the most important goals of any undergraduate course, in mathematics as in any other subject, is to foster independent critical thinking on the part of the student. A second goal is to convey some idea, appropriate to the level of the course, of the rather unusual epistemology that makes mathematics quite different from other fields of study, and some of the benefits and drawbacks of this approach to what constitutes knowledge. In contrast, I consider specific technical knowledge of the mathematical content of the course to be primarily a vehicle for achieving the first two goals.

While teaching an introductory differential equations course this quarter to students who have studied linear algebra, I have been heavily emphasizing the links between linear differential equations and linear algebra. In particular, I have been explaining how the solutions from finite dimensional vector spaces, which, unlike the spaces $\mathbb{R}^n$, have no obvious preferred basis, and how evaluating a solution and its derivatives at a point gives a linear transformation to $\mathbb{R}^n$. I am hoping this helps students see why they have studied vector spaces and linear transformations abstractly rather than just studying $\mathbb{R}^n$ and matrices. I am also hoping that the students can get some hint that this connection allows people studying linear differential equations to transfer some of their geometric intuition about vector spaces into intuition about solutions to differential equations.

In contrast, I am spending relatively less time on actually solving differential equations. While it is important for students to see concrete solutions to actual problems and have practice applying the theory in specific cases, anyone who has a practical need for solving an actual differential equation can always learn to use Matlab or Maple, especially if they already understand the theory. Similarly, when I have taught differential calculus, I have emphasized the geometric meaning of derivatives. I also point out to students that the rules for taking derivatives allow us to make conclusions for specific values with limited information; for example the product rule allows us to find $fg(2)$ from $f(2)$, $f'(2)$, $g(2)$, and $g'(2)$, without knowing any general formula for f and g. I believe this forces students to

grapple with the meaning of these rules more than applying them to functions defined by a formula.

As part of exposing students to the nature of mathematics I try to give the students some exposure to mathematical research in every course I teach. In a lower division course, I generally spend a lecture at some point explaining in general terms a problem I am thinking about. The purpose is to give students an example of research in pure mathematics. In my course on convex geometry, I spent two weeks discussing recent research on the number of faces of various dimensions in 4-dimensional polytopes. In my opinion this topic provides an accurate picture of what research in pure mathematics is frequently like. We discussed both abstract theorems giving bounds on how many faces are possible and actual constructions of polytopes which achieve these bounds. I also pointed out that any complete characterization on possible numbers of faces is likely to be horrendously complicated, so researchers are trying to find out what partial results can actualy be concisely stated and proven.

Last summer I worked with four undergraduate students on research projects. One student worked on a problem about drawings of certain particular graphs (the kind with vertices and edges) and completely solved some small cases as well as hinting at a general solution. The other three students worked on a fifteen-year-old conjecture that certain finite sets formed bases for certain particular finite-dimensional vector spaces of polynomials. I had a new approach to this old conjecture and the students explored this approach using computations with Maple, but unfortunately this approach proved to be no easier than previous approaches. With my research students over the summer I took a much less formal approach to teaching them any mathematics they needed to know. I merely gave them a broad outline of what they needed and suggested examples to work through, and left them to look up details or to ask me further questions the next day when they needed help. This approach frequently allowed them to discover the need for various definitions and theorems before actually encountering them in a textbook.

While the students I worked with over the summer have far more ability and background in mathematics than most students in a lower division course, I am finding that it occasionally works to ask students to read material from the textbook on their own, with some guidance beforehand and opportunities to ask questions afterwards. Unfortunately for reasons of time, I have to cover most of the material by lecturing, but I find that students who are persuaded to engage with the material partly on their own learn it better. Furthermore, the questions they ask tell me how they are confused, so that I do not have to guess. I welcome the occasions when a class becomes a dialogue between myself and the all the students, or, if the class is too large, the first few rows. I also encourage students to ask me questions individually or in small groups, either after class, during my office hours, by appointment at other times, or even dropping in when I am likely to be in my office.

Though I have no experience doing so, I would be interested in teaching a course where students learn to read and write proofs. I enjoy writing and editing, and believe many techniques generally used to teach writing would be useful in such a course. In addition to standard undergraduate courses, I am also particularly interested in teaching advanced

courses on topics not always found in the undergraduate curriculum. Possibilities include courses on graph theory or combinatorics, or on symmetric functions and the representation theory of the symmetric group. Although the latter is usually first taught at the graduate level, it could easily be made accessible to undergraduates who have learned linear algebra and basic group theory.

Since I am now only teaching my seventh course on my own, I still have much to learn about teaching. I hope to continue to improve as a teacher by teaching and reflecting on my teaching experiences, and through discussions with colleagues and feedback from students. While I enjoy doing research, I consider teaching the more important part of my professional life. Especially considering my interests are well within pure mathematics, I find my research not sufficiently clearly useful to society to make it the main focus of my career. As a teacher of mathematics, one has the opportunity to help many students become better at thinking logically, detecting fallacies, and communicating effectively. I look forward to teaching mathematics for many years to come.