A KINEMATIC APPROACH TO
HORIZONTAL CURVE TRANSITION DESIGN

by

James A. Bonneson, P.E.

Associate Research Engineer
Texas Transportation Institute
Texas A&M University
College Station, TX  77843-3135

(409) 845-9906
fax: (409) 845-6254
j-bonneson@tamu.edu

Submitted for presentation at the 79th Annual
Meeting of the Transportation Research Board
Washington, D.C.

November 7, 1999
ABSTRACT

Research has shown that vehicles shift laterally in the traffic lane during their entry to (or exit from) a horizontal curve. In addition, research indicates that the majority of drivers momentarily adopt a path radius that is sharper than that of the roadway curve. This research was undertaken to investigate the causes of lateral shift and sharp path radii and to determine if they can be minimized (or eliminated) using alternative horizontal curve transition design element values.

Based on a review of the driver/vehicle control process, it was concluded that the observed lateral lane shift is due to unbalanced lateral accelerations that act on the vehicle as it enters the curve. These accelerations result from gravity, as effected through roadway superelevation, and side friction due to the steer angle of the vehicle. The latter acceleration was found to be introduced through a “ramp” steer behavior. This behavior was incorporated into kinematic models of lateral acceleration, velocity, and shift. Subsequent to their calibration, these models were used to develop guidelines regarding superelevation rates and locations for the superelevation runoff section (relative to the start of the curve) that minimize lateral velocity and shift.

Key Words: geometric design, horizontal curve, vehicle control

ACKNOWLEDGMENT

This work was sponsored by the American Association of State Highways and Transportation Officials, in cooperation with the Federal Highway Administration, and was conducted in the National Cooperative Highway Research Program (NCHRP) which is administered by the Transportation Research Board of the National Research Council. The materials and methods presented were developed as part of NCHRP Project 15-16 “Superelevation Distribution Methods and Transition Designs.”
A KINEMATIC APPROACH TO
HORIZONTAL CURVE TRANSITION DESIGN

INTRODUCTION

Overview and Objective

The horizontal curve transition section, located near the curve’s beginning and ending points, is used to facilitate a safe and comfortable curve entry or exit. The transition section consists of two design components: (1) superelevation transition and (2) alignment transition. The first component provides for the pavement’s rotation from a normal cross slope rate to a fully superelevated rate. The second component provides for the gradual introduction of curvature by the use of a transition curve (e.g., a spiral curve) located between the tangent and horizontal curve.

Several researchers (1, 2, 3, 4) have studied the effect of curve transition design on a vehicle’s lateral placement during curve entry or exit. All have reported observing a tendency for the vehicle to shift laterally; with shifts of 1.0 m or more found on sharper curves. Some researchers believe this shift is due to the driver’s desire to flatten their path radius (i.e., cut the corner); however, it is also possible that the shift is due to unbalanced lateral accelerations resulting from superelevation and steering. Regardless, shifts in excess of 1.0 m are undesirable as it is likely that they are associated with vehicle encroachment into the adjacent lane or shoulder.

Several researchers (2, 3, 4) have found that drivers vary their path radius throughout the curve; however, their collective findings also indicate that the sharpest path radii occur near the beginning and the end of the curve. The smallest (or sharpest) path radius that momentarily occurs along the curve is defined herein as the “critical” path radius. Glennon et. al. (4) reported that the 50th percentile driver’s critical path radius is about 88 percent of the roadway curve’s radius (i.e., the critical path radius is sharper than that of the curve). The adoption of a path radius sharper than that of the curve is undesirable as it may result in a peak side friction demand that exceeds that intended by the curve’s designer.

The objective of this research was to investigate the causes of lateral shift and critical path radii and to determine if they can be minimized (or eliminated) using alternative transition design element values. Transition design elements are described in the 1994 AASHTO publication A Policy on Geometric Design of Highways and Streets (5) (referred to herein as the Green Book); they include the design superelevation rate and the portion of runoff located prior to the curve.

This paper focuses on the “tangent-to-curve” transition design. The term “tangent-to-curve”is used herein to refer to the situation where compound or spiral curvature is not used in the transition. Thus, in the tangent-to-curve design, the tangent section of the alignment intersects directly with the horizontal curve.
Background

Vehicle Control - The Driving Process

Numerous researchers (6, 7, 8) have described the driving process in the form of a real-time, closed-loop vehicle control model. This type of model uses anticipatory and compensatory response mechanisms to simulate driver behavior. The anticipatory mechanism serves the vehicle guidance function in that it uses visual input about road conditions ahead to prepare and initiate appropriate vehicle control inputs (e.g., steer angle). This mechanism is intended to avoid errors in lane position, speed choice, or vehicle direction. The selection of an appropriate control input in response to this information is based largely on the driver’s recollection of similar geometric conditions and successful responses (i.e., the driver’s expectancy).

The compensatory mechanism serves the vehicle control function in that it uses information about current vehicle lane position, speed, heading angle, and lateral acceleration to continuously revise the vehicle control inputs. This mechanism is intended to minimize the magnitude of errors (or undesirable deviations) in vehicle control once it occurs. Information for this mechanism is based on visual and kinesthetic sensory inputs. The control response to this information is based on the driver’s understanding of their vehicle’s performance in terms of the effect of changes in steering and speed control on vehicle position, heading angle, and stability.

Both Donges (6) and Godthelp (7) applied their vehicle control models to the study of driver behavior during curve entry. They calibrated their models by observing subject drivers negotiate a test course with curves of various radii and direction. Both researchers concluded that drivers start the steering action a short time before the curve begins (i.e., the PC) and end a short time after the PC. This time interval was defined as the “anticipatory time” $t_a$ as it relates to the anticipatory response mechanism.

Driver Steering Behavior During Curve Entry

Figure 1 describes the relationship between road geometry, curvature, and steering wheel angle during the curve entry process. The trends shown indicate that the driver initiates the steer $t_a$ seconds prior to the PC. Thereafter, the steering-wheel angle increases at a constant rate until the angle needed to negotiate the curve $\delta_i$ is reached. This steer maneuver is referred to as a “ramp steer” and its duration is referred to as the “steering time” $t_s$. The steering oscillations that follow the ramp slope result from the compensatory response mechanism as the driver attempts to stabilize the vehicle’s lateral motion and adopt a circular, path-following driving mode.

Donges (6) measured the anticipatory time during a series of experiments using a driving simulator. He found that anticipatory time $t_a$ averaged 1.1 s and that it was insensitive to vehicle speed. Donges also provided a plot of measured steering wheel angle versus time (similar to that shown in Figure 1). Examination of this plot indicates that the ramp steer input continues beyond the PC for a time approximately equal to $t_a$. This finding suggests that the steering time is approximately equal to twice the anticipatory time (i.e., $t_s \approx 2t_a$). Similar findings were also reported by Stewart (9).
The mechanics of a vehicle’s steering system produces the following relationship between steering wheel angle and path curvature:

\[ c(t) = \frac{1000 \delta(t)}{r_s L (1 + 0.00199 v^2)} \]  

(1)

where:

- \( c(t) \) = curvature of the travel path at time \( t \) (= 1000/R(t)), \( \text{km}^{-1} \);
- \( \delta(t) \) = steering-wheel angle;
- \( r_s \) = steering-wheel radius;
- \( L \) = length of the vehicle;
- \( v \) = vehicle speed, \( \text{km/h} \);
- \( R(t) \) = traffic lane radius at time \( t \).

Variable definitions:
- \( t_b \) = time that vehicle enters curve,
- \( t_e \) = time that vehicle exits curve,
- \( R_p \) = traffic lane radius,
- \( c_r \) = traffic lane curvature (= 1/\( R_p \)),
- \( \delta_s \) = steering-wheel angle,
- \( t_a \) = anticipatory time.

Figure 1. Steering wheel angle during curve entry (adapted from Ref. 7).
\[ R(t) = \text{instantaneous radius of the travel path at time } t, \text{ m}; \]
\[ \delta_i(t) = \text{steering-wheel angle at time } t, \text{ rad}; \]
\[ r_s = \text{steering wheel to front wheel angle ratio (typically about 20:1 for passenger cars)}; \]
\[ L = \text{wheelbase, m}; \text{ and} \]
\[ v = \text{vehicle speed, m/s}. \]

This equation indicates that there is a linear relationship between steering wheel angle and curvature. A unit change in steering wheel angle produces a unit change in curvature. A constant rate of change in angle over time (or travel distance) produces a corresponding constant rate of change in curvature. As the spiral shape represents a constant rate of change in curvature, the ramp steering behavior produces a spiral travel path. This point is noted by the Green Book authors (5, p. 174) and is also discussed by Stewart (9) and by Glennon et al. (4).

Glennon et al. (4) measured the path curvature of passenger cars entering several two-lane, rural highway curves. The typical travel path curvature observed by Glennon is shown in Figure 2.

![Figure 2. Relationship between roadway curvature and travel path curvature.](image-url)

The data reported by Glennon et al. (4, p. 141) suggest that 52 percent of all drivers deviate from the typical travel path curvature (as shown in Figure 2) by adopting, at some point after the curve PC, a critical path radius that is sharper than that of the roadway. This steer behavior is depicted in Figure 2 by the thin line oscillating about the desired roadway curvature. In the context of the vehicle control model described previously, the critical radius is likely a result of steering adjustments during the compensatory stage of the curve negotiation process.
MODEL DEVELOPMENT

Model Framework

This section describes a kinematic model for predicting a vehicle’s lateral motion while traveling through a transition section. In this regard, a “point-mass” representation of the vehicle is used; such a representation is generally suitable for modeling non-articulated vehicles on roadway curves. A more complicated, vehicle dynamics model would be needed to precisely model tractor-trailer trucks and other articulated vehicles. The model described herein is developed for the tangent-to-curve transition design. A second model of lateral motion was developed for the spiral transition design. A complete description of this latter model is provided in Reference 10.

The lateral motion model developed herein consists of two main equations. One equation is used to predict the vehicle’s lateral velocity at the end of the transition section and the other is used to predict its lateral shift distance. These models are intended to provide a first-order approximation of the vehicle’s lateral velocity and shift. They are based on an idealized ramp steer behavior and its associated duration (i.e., steering time). The model predictions are intended to be sufficiently accurate to estimate the relative merits of alternative transition design element values.

The lateral motion model is not intended to be a precise predictor of lateral velocity or shift as it is recognized that steer behavior is more complicated than can be described by the ramp steer model. Rather, the ramp steer behavior is believed to represent a “desirable” steering response. Thus, the model can be used to identify geometric conditions that enable drivers to reproduce a desirable steering response and thereby, minimize the need for corrective steering oscillations (and adoption of a critical path radius).

The lateral motion model is based on the lateral accelerations acting on the vehicle as it traverses the transition section. In general, there are two sources of lateral acceleration acting on the vehicle in this section: (1) acceleration due to gravity and (2) acceleration due to tire-pavement friction. The first acceleration results from roadway superelevation and the second results from steer input. These accelerations tend to be equal and opposite prior to the transition and combine to equal the centripetal acceleration of the curve after the transition. The variation of these accelerations through the transition section tends to result in lateral motion. These accelerations are shown in Figure 3 for a curve to the right.

Sign Convention and Assumptions

Acceleration relationships, similar to those shown in Figure 3, were also developed for a curve to the left. These relationships indicated that the same model form was suitable for both curve directions, provided that a sign convention was adopted that recognized the directional relationships of the corresponding accelerations. Thus, one model was developed such that the signs of all variables are positive when applied to the right-hand curve. When applied to a left-hand curve, the design superelevation rate, radius of curve, maximum relative gradient, and minimum runout length should be assigned negative values by the analyst.
Figure 3. Lateral accelerations during entry to a right-hand curve with a tangent-to-curve transition design.
In addition to the aforementioned sign convention, the following assumptions were made in developing the tangent-to-curve model:

1. The driver exerts whatever steering effort is needed to counter the normal crown cross slope and maintain a constant lane position up until $t_a$ seconds prior to the PC. At this point in time, the driver is assumed to initiate the ramp steer behavior for $t_s$ seconds (which is equal to $2t_a$).

2. Curve length exceeds the distance traveled during steering time.

3. The superelevation runoff length is equal to the minimum length recommended in the *Green Book* (5), as computed using the following equation:

$$L_r = \text{larger of:} \left\{ \begin{array}{c} \frac{we_d n_l b_w}{\Delta} \\ \frac{2V_d}{3.6} \end{array} \right\} \tag{2}$$

where:
- $L_r =$ minimum length of superelevation runoff, m;
- $\Delta =$ maximum relative gradient (*Green Book* (5) Table III-13), percent;
- $b_w =$ adjustment factor for number of lanes rotated (desirable minimum $b_w = 1.0$, absolute minimum $b_w$ equals 1.0, 0.80, 0.75 and 0.67 for $n_l$ equal to 1.0, 1.5, 2.0, and 3.0);
- $w =$ width of one traffic lane (typically 3.6 m), m;
- $e_d =$ design superelevation rate, percent;
- $V_d =$ design speed, km/h; and
- $n_l =$ number of lanes rotated, lanes.

4. The tangent runout length equals that recommended in the *Green Book* (5), as computed using:

$$L_t = \frac{e_{NC}}{e_d} L_r \tag{3}$$

where:
- $L_t =$ minimum length of tangent runout, m; and
- $e_{NC} =$ normal crown cross slope rate (typically 2.0 %.), percent.

The runoff length obtained from Equation 2 can be used to determine the effective maximum relative gradient. This gradient incorporates the variable $b_w$ and reflects the length used regardless of which component of Equation 2 controls. The effective gradient can be computed as:
J.A. Bonneson  
Texas Transportation Institute

\[
\Delta^* = \frac{w e_d}{L_r} n_l \tag{4}
\]

where:
\( \Delta^* \) = effective maximum relative gradient, percent.

**Lateral Acceleration due to Superelevation**

The acceleration due to superelevation can be computed as:

\[
a_e(x) = g e(x) 0.01 \tag{5}
\]

with,

\[
e(x) = \begin{cases} 
  e_{NC} & : x \leq x_1 \\
  (e_d - e_{NC}) \frac{x - x_1}{x_3 - x_1} + e_{NC} & : x_1 < x < x_3 \\
  e_d & : x \geq x_3
\end{cases} \tag{6}
\]

\[
x_1 = x_{PC} - (P_r L_r - L_r) \tag{7} \quad x_3 = x_{PC} + (1 - P_r) L_r \tag{8}
\]

where:
\( a_e(x) \) = acceleration sustained by superelevation at a distance \( x \) along the transition, m/s\(^2\);
\( g \) = gravitational acceleration (= 9.807 m/s\(^2\));
\( e(x) \) = superelevation rate at a distance \( x \) along the transition section, percent;
\( P_r \) = portion of superelevation runoff located prior to the curve;
\( x_1 \) = location where superelevation begins its change from \( e_{NC} \) to \( e_d \) relative to the PC, m;
\( x_3 \) = location where superelevation ends its change from \( e_{NC} \) to \( e_d \) relative to the PC, m; and
\( x_{PC} \) = location of the curve beginning (defined in Figure 3 as 0.0), m.

**Lateral Acceleration due to Friction**

The acceleration due to tire-pavement friction that results from the ramp steer behavior can be computed as:
\[ a_f(x) = \begin{cases} -e(x) g 0.01 & : x \leq x_a \\ \left( \frac{v^2}{R_p} - \frac{e_d}{100} g + \frac{e_s}{100} g \right) \frac{x - x_a}{x_b - x_a} - \frac{e_s}{100} g & : x_a < x < x_b \\ \frac{v^2}{R_p} - \frac{e_d}{100} g & : x \geq x_b \end{cases} \] (9)

\[ e_s = \begin{cases} (e_d - e_{NC}) \frac{x_a - x_1}{x_2 - x_1} + e_{NC} & : x_a > x_1 \\ e_{NC} & : x_a \leq x_1 \end{cases} \] (10)

\[ R_p = R - w (n_l - 0.5) \] (11)

\[ x_b = x_{PC} + \frac{t_s}{2} v \] (12)

\[ x_a = x_{PC} - \frac{t_s}{2} v \] (13)

The accelerations resulting from superelevation and steering must combine to provide the centripetal acceleration required to track the traffic lane, as shown in the bottom portion of Figure 3. The difference between the applied lateral accelerations and the centripetal acceleration equals the acceleration available for lateral motion. This resultant lateral acceleration can be computed as:
\[ a_l(x) = a_c(x) + a_f(x) - a_r \]  \hspace{1cm} (14)

where:
\( a_l(x) \) = resultant lateral acceleration at a distance \( x \) along the transition, \( \text{m/s}^2 \); and
\( a_r \) = centripetal acceleration \( (= \frac{v^2}{R_p} \text{ if } x > x_{PC}; \text{ otherwise } = 0.0) \), \( \text{m/s}^2 \).

Equations 5 through 14 were applied to a typical two-lane highway curve having a radius of 249 m and a superelevation rate of 8 percent. The speed was assumed to be 61 km/h and the steering time was assumed to be 2.8 s. The resulting lateral accelerations are shown in Figure 4.

The trend line shown in Figure 4 indicates that the resultant lateral acceleration (i.e., the thick line) is initially equal to zero as the acceleration due to friction is equal and opposite to that required to maintain lane position on a normal crown section. The driver initiates the ramp steer at 1.4 s (24 m) before the PC. At 21 m before the PC, the superelevation transition section is encountered and additional acceleration due to gravity is introduced. The resultant acceleration increases to its maximum value just prior to the PC as the steering wheel angle and superelevation rates are gradually increased. As a result of these two accelerations, the vehicle drifts to the right.

After the PC, the centripetal acceleration required by the curve is large and not fully matched by the combined steer and superelevation-related accelerations. The resultant acceleration is also...
large and in a direction opposite to that experienced prior to the PC. As a result, the vehicle drift to the right begins to slow. As the vehicle moves further along the transition, the applied accelerations continue to increase until they match that required to track the curve radius. At this point (+24 m), the drift is significantly slowed (and possibly stopped).

**Lateral Velocity**

Lateral velocity at any point can be computed by the integration of acceleration over time or distance. It was determined that integration over distance would be most helpful in satisfying the objectives of this research as many of the transition elements are represented by length along the roadway rather than travel time. Thus, the integral had the following basic form:

\[
v_l(x) = \frac{1}{v^2} \int a_f(x) \, dx
\]  

(15)

where:

\( v_l(x) = \text{lateral velocity at a distance } x \text{ along the transition, m/m.} \)

The lateral velocity obtained from this equation represents meters of lateral shift for each meter of forward progress. Hence, it has units of meters per meter (i.e., m/m).

Figure 5 illustrates the lateral velocity resulting from the example curve described for Figure 4. The thick trend line shown indicates that lateral velocity is positive denoting a drift to the right (inward). This drift reaches its maximum value at the PC and then slows to a small positive quantity. This non-zero lateral velocity at the end of the transition would require a small steer correction by the driver or inward drifting will continue.

The thin trend line shown in Figure 5 illustrates the effect of placing more of the superelevation runoff on the curve (i.e., \( P_r = 0.50 \)). The result is that there is less lateral acceleration prior to the PC and more thereafter. In fact, the combined steer and superelevation accelerations are so large after the PC that they induce a negative (or outward) drift. This non-zero drift will require a small steer correction to negate. More importantly, the outward direction of drift suggests that the correction required will be one of increased steer angle which will create a critical path radius that is smaller than that of the highway curve.
Figure 5. Lateral velocity during curve entry.

While knowledge of the lateral velocity at various points along the transition is useful, knowledge of this velocity at the end of the transition section is the key to understanding the effect of alternative transition designs. In recognition of this fact, Equation 15 was integrated to yield the following equation for predicting lateral velocity at the end of the transition:

\[
\begin{align*}
v_f &= \begin{cases} 
\frac{g}{2} \left[ 0.01 \frac{\Delta^*}{v^2 w n_l} \left( 0.5 t_s v^2 - L_r^2 (1 - P_r)^2 \right) \right] & : P_r > \frac{0.5 t_s v + L_l}{L_r} \\
\frac{g}{2} \left[ 0.01 \frac{\Delta^*}{v^2 w n_l} \left( P_r L_r - L_l \right)^2 - L_r^2 (1 - P_r)^2 \right] & : P_r \leq \frac{0.5 t_s v + L_l}{L_r}
\end{cases}
\end{align*}
\]

where:

\( v_f \) = lateral velocity the end of the transition, m/m.

The two forms of the equation are necessitated by the independence of the two applied accelerations. In this regard, the first equation is appropriate when the ramp steer is initiated after the superelevation rotation begins. In contrast, the second equation applies when the ramp steer is initiated prior to the point where superelevation rotation begins. It should be noted that the resultant lateral velocity predicted by Equation 16 is applicable to either curve entry or exit due to the integration process.
Lateral Shift

Lateral shift at any point can be computed by the integration of velocity over time or distance. As before, it was determined that integration over distance would be most helpful in satisfying the objectives of this project. Thus, the integral had the following basic form:

\[ y_l(x) = \int v_l(x) \, dx \]  

(17)

where:

\[ y_l(x) = \text{lateral shift at a distance } x \text{ along the transition, m.} \]

Figure 6 illustrates the lateral shift resulting from the example curve described for Figure 4. The thick trend line shown indicates that the lateral shift is positive denoting a drift to the right (inward). The shift reaches its maximum value of about 0.42 m at the end of the steering time (as identified by point A).

The thin trend line shown in Figure 6 illustrates the effect of placing more of the superelevation runoff on the curve. The combined steer and superelevation accelerations are so large after the PC that they induce a negative (or outward) drift that returns the vehicle to nearly the same lateral position it had prior to curve entry (as identified by point B).
While knowledge of the lateral shift at various points along the transition is useful, knowledge of this shift at the end of the transition section is key to understanding the effect of alternative transition designs. In recognition of this fact, Equation 17 was integrated to yield the following equation for predicting lateral shift at the end of the transition:

\[ y_I = y_s + y_g - y_R \]  \hspace{1cm} (18)

with,

\[
y_s = \left[ \frac{1}{R_p} - \frac{g \Delta^*}{v^2 w n_l} \left( x_p \right) \right] \left( \frac{x_s^2}{2} + \frac{t_s v x_s}{2} + \frac{(t_s v)^2}{6} \right) \]  \hspace{1cm} (19)

\[
y_g = \frac{g \Delta^*}{v^2 w n_l} \left( x_p \right) \left( \frac{x_g^2}{2} + \frac{x_p x_g}{2} + \frac{x_p^2}{6} \right) \]  \hspace{1cm} (20)

\[
y_R = \frac{x_b^2}{2 R_p} \]  \hspace{1cm} (21)

\[
x_p = \begin{cases} 
0.5 t_s v + L_r (1 - P_r) & : P_r > \frac{0.5 t_s v + L_t}{L_r} \\
L_r - L_t & : P_r \leq \frac{0.5 t_s v + L_t}{L_r}
\end{cases} \]  \hspace{1cm} (22)

\[ x_b = \text{larger of:} \left[ 0.5 t_s v, \ L_r (1 - P_r) \right] \]  \hspace{1cm} (23)

\[ x_s = x_b - 0.5 t_s v \]  \hspace{1cm} (24)

\[ x_g = x_b - L_r (1 - P_r) \]  \hspace{1cm} (25)

where:

\[ y_I = \text{lateral shift the end of the transition, m.} \]
The two variations within Equation 22 are necessitated by the independence of the two applied accelerations. In this regard, the first equation is appropriate when the ramp steer is initiated after the superelevation rotation begins. In contrast, the second equation applies when the ramp steer is initiated prior to the point where superelevation rotation begins. It should be noted that the resultant lateral shift predicted by Equation 18 is applicable to either curve entry or exit due to the integration process.

MODEL CALIBRATION

The model calibration step consisted primarily of defining the value of steering time \( t_s \) that most accurately reproduced observed lateral shifts. This lateral shift data was obtained from the report by Segal et al. (2). They measured the lateral shift of passenger cars and trucks at 6-m intervals along two curves. These curves had 46 and 55-m radii and were located on interchange off-ramps. The lateral shifts of 30 passenger cars and 30 trucks were observed on each curve. Data describing the superelevation rate at 13 to 16 points along the transition section of each curve location were also reported by Segal.

The model calibration consisted of obtaining a visual “best fit” between the predicted and the average of the observed lateral shifts. The model calibration parameter was steering time \( t_s \). Once the speed, radius, and superelevation information were input to the model, the steering time parameter was varied until the best fit was obtained for the entire shift trace. This process was repeated for each of the two vehicle types and for each of the two curves studied by Segal et al. (2).

A comparison of the observed and predicted lateral shifts at one curve are shown in Figure 7. The fit was similar for the other site. In general, the model fit to the observed data was believed to be quite good given the simplicity of the assumed ramp steer model. Steering time values for the passenger cars ranged from 2.40 to 3.04 s; those for the trucks were slightly higher at about 3.70 s. Based on this comparison and the findings reported by others (3, 4, 6, 7), a steering time value of 2.8 s was selected as being representative of most passenger cars.

SENSITIVITY ANALYSIS

This section describes an examination of the effect of superelevation rate, design speed, and runoff location on lateral motion. Runoff location was evaluated in terms of the portion of runoff located prior to the curve. The Green Book (5, p. 181) does not define a specific value for this design element; however, it does indicate that locating 0.5 to 1.0 of the runoff prior to the curve is “suitable” and that 0.6 to 0.8 is “desirable.”

Equations 16 and 18 were used to compute the lateral velocity and shift for each of the three aforementioned design variables. Lateral velocity and shift were computed for the right-hand (i.e., inside) and left-hand (outside) curve directions. Curve radius was computed from a superelevation distribution relationship similar to Distribution Method 5 (as described in the Green Book (5)) for each combination of design speed and superelevation rate considered.
Three points need to be made regarding the interpretation and assessment of lateral velocity and shift. First, the lateral velocity and shift for both curve directions should be considered together as most roadways serve two-way traffic flows. In this regard, acceptable values of velocity or shift are necessary for both directions when assessing the merits of a specific design element value.

Second, lateral velocities near zero at the end of the transition are most desirable; however, they are not achievable in both travel directions for most variable combinations. Hence, small lateral velocities should be considered acceptable provided that they are in an “inward” direction (i.e., positive for the inside direction and negative for the outside direction). As mentioned previously, an outward velocity will likely require the driver to adopt an undesirable critical path radius. Based on an examination of predicted velocities for a range of typical design values, inward lateral velocities up to 0.01 m/m were considered acceptable.

Third, the lateral shift should not be excessively large. Large shift values will likely result in the vehicle encroaching on an adjacent lane or shoulder. In this regard, predicted shift values of up to 1.0 m were considered acceptable for lane widths of 3.0 to 3.6 m. It should be noted that this value of shift would accommodate about 90 percent of the passenger cars observed by Glennon et al. (4).

**Effect of Superelevation Rate and Speed**

The results of the evaluation of superelevation rate and speed are shown in Figure 8. The minimum runoff length $L_{r,min}$ shown represents two seconds travel time at the design speed (i.e., the
second component of Equation 2). This evaluation revealed that slower speeds and larger superelevation rates are associated with large, outward velocities. Similar trends were found with regard to lateral shift. Although these trends are the result of several factors in combination, they are mainly due to the fact that runoff length exceeds steering time (represented in terms of travel distance, i.e., $t_s v$) when speeds are low and superelevation rates are large.

![Figure 8. Effect of superelevation rate and speed on lateral velocity.](image)

Further examination of these two variables indicated that a “critical” superelevation rate existed for each design speed. A superelevation rate in excess of this critical (or limiting) rate was found to yield lateral shifts in excess of 1.0 m and outward lateral velocities in excess of 0.01 m/m. These limiting superelevation rates were found to be 8.2, 9.8, 10.8, 11.4, and 11.8 percent for design speeds of 30, 40, 50, 60, and 70 km/h, respectively. The limiting rate was larger than 12 percent for designs speeds of 80 km/h and above and thus, poses no practical concern.

**Effect of Runoff Location**

Equation 16 was used to examine the relationship between the portion of superelevation runoff located prior to the curve and lateral velocity. For this examination, a two-lane highway curve with a superelevation rate of 6.0% was considered as was entry to the curve from both directions. The results of this analysis are shown in Figure 9.
Figure 9. Effect of portion-of-runoff-prior-to-curve on lateral velocity.

The trends shown in Figure 9 indicate that portion does not have the same effect on lateral velocity in the two travel directions. For the left-hand direction, lateral velocity decreases with increasing portion. It reaches a desirable lateral velocity of zero when the portion is about 0.3. For the right-hand direction, lateral velocity increases with increasing portion. It reaches a desirable velocity of zero when the portion is about 0.67.

Two line thicknesses are used in Figure 9 for each of the trend lines. The part of the line that is thin denotes an undesirable outward lateral velocity, relative to the travel direction. The thick part of the line denotes an acceptable inward lateral velocity. Based on this identification, a portion of 0.67 would appear to offer the best compromise value for the conditions analyzed. This portion should have negligible drift in the inside direction and an inward drift in the outside direction that is very near the acceptable limit (i.e., 0.01 m/m). Portions above or below the value of 0.67 produce either undesirable outward drift or undesirably large lateral velocities.

The trends shown in Figure 9 correspond to a vehicle traveling at 70 km/h. As a design should conservatively embrace the distribution of speeds found on a roadway, the portion of runoff located prior to the curve should adequately serve both the slow (say, 5th percentile) and fast (say, 95th percentile) drivers. Additional analysis using the Equations 16 and 18 led to the conclusion that a portion can be identified that minimizes lateral velocity and shift for the large majority of drivers. This “optimum” value was found to vary from 0.8 at 30 km/h to 0.7 at 120 km/h. It also tended to increase by 0.1 for each additional lane included in the rotated section of pavement. Additional discussion on the development of these optimum values is provided in Reference 10.
CONCLUSIONS

Several conclusions were reached as a consequence of this research. These conclusions relate to the vehicle control process, the lateral motion model, and the identification of appropriate transition design element values.

The literature review of driver steer behavior indicates that drivers initiate their steer based on their perception of curve location. The break in alignment at the point of curvature (i.e., the PC) is a key piece of information available to the driver’s anticipatory response mechanism. However, this apparent benefit of a tangent-to-curve transition is not generally acknowledged in the field of highway design. In fact, the AASHTO Green Book (5) indicates that one of the principal advantages of a spiral transition is that it “...avoids the noticeable breaks at the beginning and ending of circular curves [that use the tangent-to-curve design]...”(5, p. 175).

Based on the results of the model calibration, it is concluded that the ramp steer model represents a reasonable, first-order approximation of driver steering behavior. It is also concluded that the lateral motion model (which is based on the ramp steer model) is sufficiently accurate to define acceptable design element values. Finally, it is concluded that the model is accurate for a wide range of conditions when a steering time of 2.8 s is used.

Based on the results of the sensitivity analysis, there appear to be some design element values that can reduce lateral velocity and shift. Specifically, it appears that superelevation rates in excess of 8.2, 9.8, 10.8, 11.4, and 11.8 percent for design speeds of 30, 40, 50, 60, and 70 km/h, respectively, should be avoided in design as they are likely associated with undesirable lateral shifts (i.e., shifts in excess of 1.0 m).

The portion of runoff prior to the curve appears to have a significant effect on lateral velocity. Optimum values of this variable were identified that resulted in minimal lateral shifts and velocities for the distribution of drivers in both travel directions. The optimum values varied from 0.8 at 30 km/h to 0.7 at 120 km/h. They tended to increase by 0.1 for each additional lane included in the rotated section of pavement.

REFERENCES


