

# Practices

Includes  
Professional  
Development  
Guide

for  
Orchestrating  
Productive  
Mathematics  
Discussions

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NATIONAL COUNCIL OF  
TEACHERS OF MATHEMATICS



# 5 Practices for Orchestrating Productive Mathematics Discussions

**Anticipating** students' solutions to a mathematics task

**Monitoring** students' in-class, "real-time" work on the task

**Selecting** approaches and students to share them

**Sequencing** students' presentations purposefully

**Connecting** students' approaches and the underlying mathematics

These 5 manageable practices have the power to put teachers in control of productive classroom discussions.

## **Read what educators are saying:**

*5 Practices for Orchestrating Productive Mathematics Discussions* provides teachers with concrete guidance for engaging students in discussions that make the mathematics in classroom lessons transparent to all. These instructional practices are extremely timely in light of the focus on Standards for Mathematical Practice in the Common Core State Standards for Mathematics, and they will support teachers and students in engaging in these standards. This book will serve as a valuable foundation in our upcoming professional development.

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Ensuring that students have the opportunity to reason mathematically is one of the most difficult challenges that teachers face. A key component is creating a classroom in which discourse is encouraged and leads to better understanding. Productive discourse is not an accident, nor can it be accomplished by a teacher working on the fly, hoping for a serendipitous student exchange that contains meaningful mathematical ideas. While acknowledging that this type of teaching is demanding, Smith and Stein present five practices that any teacher can use to implement coherent mathematical conversations. By using the five practices, teachers will learn to teach effectively in this way.

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