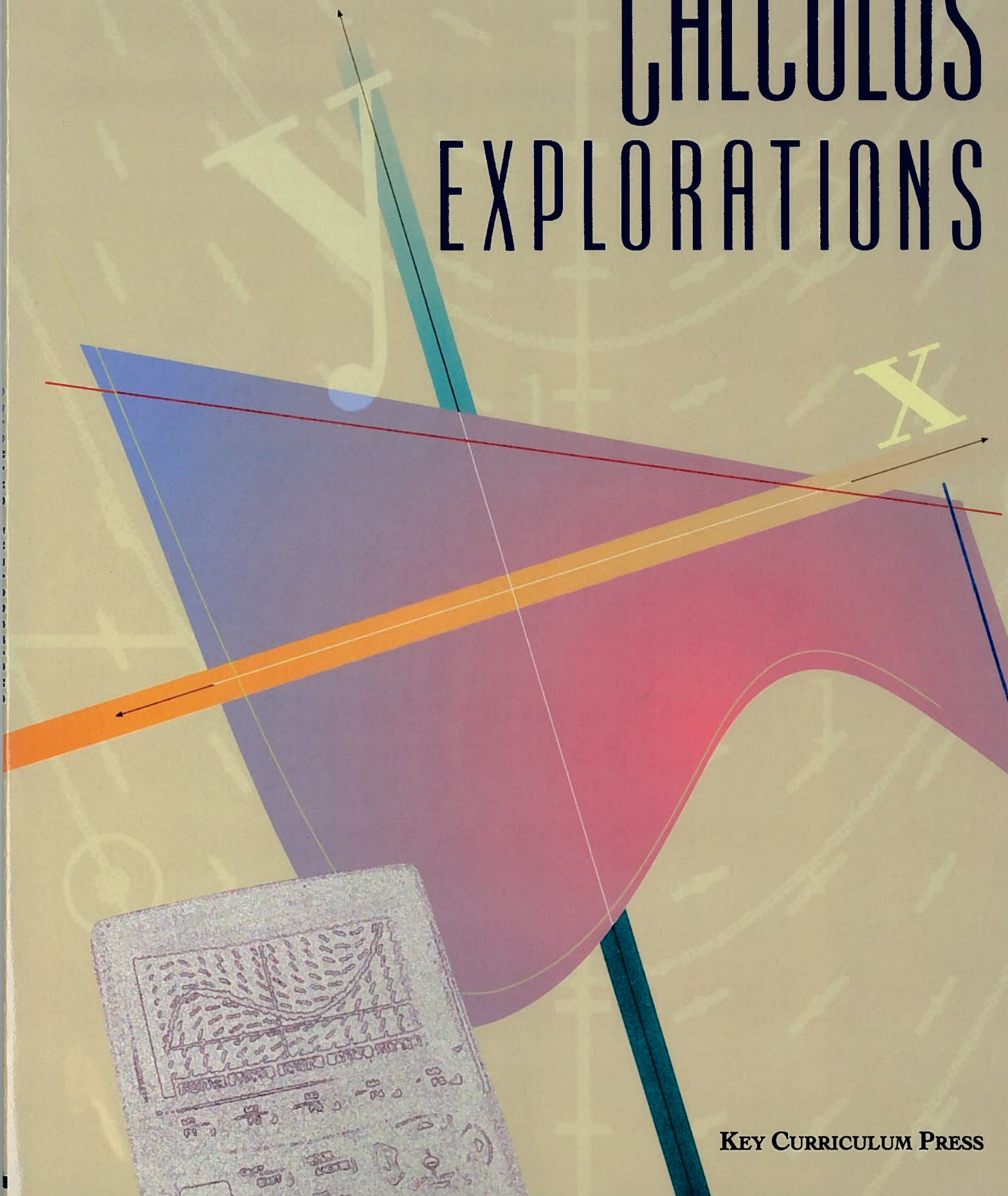


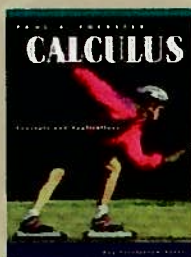
PAUL A. FOERSTER

CALCULUS EXPLORATIONS



KEY CURRICULUM PRESS

CALCULUS EXPLORATIONS



The activities in this volume help students extend their understanding of important concepts in calculus. The Explorations are designed to be worked with a graphing calculator and can be used to supplement any calculus textbook. They reinforce such topics as l'Hospital's rule, the mean value theorem, Riemann sums, and the integration of trigonometric functions, among others. Whether working individually or in cooperative groups, students will find these unique Explorations both challenging and instructive in developing their understanding of calculus. Drawn from the *Instructor's Resource Book* for Paul A. Foerster's innovative textbook, *Calculus: Concepts and Applications*, the material can be used individually by students or as blackline masters reproduced by the instructor as needed.



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