

Developing
**Essential
Understanding**
of
Functions

Grades 9–12



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Developing Essential Understanding

Are sequences functions? Why can't the popular "vertical line test" be applied in some cases to determine whether a relation is a function? How does the idea of rate of *change* connect with simpler ideas about proportionality as well as more advanced topics in calculus?

How much do you know ... and how much do you *need* to know?

Helping your high school students develop a robust understanding of functions requires that you understand this mathematics deeply. *But what does that mean?*

This book focuses on essential knowledge for teachers about functions. It is organized around five big ideas, supported by multiple smaller, interconnected ideas—*essential understandings*. Taking you beyond a simple introduction to functions, the book will broaden and deepen your mathematical understanding of one of the most challenging topics for students—and teachers. It will help you engage your students, anticipate their perplexities, avoid pitfalls, and dispel misconceptions. You will also learn to develop appropriate tasks, techniques, and tools for assessing students' understanding of the topic.

Focus on the ideas that you need to understand thoroughly to teach confidently.

Move beyond the mathematics you expect your students to learn. Students who fail to get a solid grounding in pivotal concepts struggle in subsequent work in mathematics and related disciplines. By bringing a deeper understanding to your teaching, you can help students who don't get it the first time by presenting the mathematics in multiple ways.



The Essential Understanding Series addresses topics in school mathematics that are critical to the mathematical development of students but are often difficult to teach. Each book in the series gives an overview of the topic, highlights the differences between what teachers and students need to know, examines the big ideas and related essential understandings, reconsiders the ideas presented in light of connections with other mathematical ideas, and includes questions for readers' reflection.

9-10/2.5K/VP

ISBN 978-0-87353-623-3

13483



9 780873 536233

Contents

Foreword	v
Preface	vii
Introduction	1
Why Functions?	1
Understanding Functions	2
Big Ideas and Essential Understandings	3
Benefits for Teaching, Learning, and Assessing	4
Ready to Begin	5
Chapter 1	7
Functions: The Big Ideas and Essential Understandings	
The Function Concept	12
Functions can show relationships between varying quantities	12
Definitions of <i>function</i>	13
“Single-valuedness” and the vertical line test	15
The concept of function is broad and flexible	18
Covariation and Rate of Change	23
Covariation	23
Rate of change	25
Families of Functions and Their Role in Modeling	
Real-World Phenomena	34
Linear functions	36
Quadratic functions	45
Exponential functions	53
Trigonometric functions	59
Combining and Transforming Functions	70
Adding, subtracting, multiplying, and dividing functions	70
Composition of functions	72
Composing with “translating” and “scaling”	
functions to transform graphs	74
Inverses of functions	75
Multiple Representations of Functions	78
Chapter 2	85
Connections: Looking Back and Ahead in Learning	
Functions in Middle Grades-Mathematics	85

Analyzing covariation between variables	85
Families of functions	86
Representing relationships with graphs, tables, and rules	88
Connections to Collegiate Studies	90
Differential equations	90
Extending domains of functions to the complex numbers	91
Chapter 3	93
Challenges: Learning, Teaching, and Assessing	
The Concept of Function	93
Functions as Models for Real-World Phenomena	95
Graphical and Algebraic Representations of Functions	96
Assessing Big Ideas and Essential Understandings	98
Extending problems	99
Reversing problems	102
Assessing Outcomes	103
References	107