

Developing
**Essential
Understanding**
of

Ratios, Proportions & Proportional Reasoning

Grades 6–8



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Developing Essential Understanding

How do you refute the claim that all ratios are fractions? How are ratios related to fractions? How does ratio reasoning differ from other types of reasoning? When it is appropriate to reason proportionally?

How much do you know ... and how much do you *need* to know?

Helping your middle school students develop a robust understanding of ratios, proportions, and proportional reasoning requires that you understand this mathematics deeply. *But what does that mean?*

This book focuses on essential knowledge for teachers about ratios, proportions, and proportional reasoning. It is organized around one big idea, supported by multiple smaller, interconnected ideas—*essential understandings*. Taking you beyond a simple introduction to ratios, proportions, and proportional reasoning, the book will broaden and deepen your mathematical understanding of one of the most challenging topics for students—and teachers. It will help you engage your students, anticipate their perplexities, avoid pitfalls, and dispel misconceptions. You will also learn to develop appropriate tasks, techniques, and tools for assessing students' understanding of the topic.

Focus on the ideas that you need to understand thoroughly to teach confidently.

Move beyond the mathematics you expect your students to learn. Students who fail to get a solid grounding in pivotal concepts struggle in subsequent work in mathematics and related disciplines. By bringing a deeper understanding to your teaching, you can help students who don't get it the first time by presenting the mathematics in multiple ways.



The Essential Understanding Series addresses topics in school mathematics that are critical to the mathematical development of students but are often difficult to teach. Each book in the series gives an overview of the topic, highlights the differences between what teachers and students need to know, examines the big ideas and related essential understandings, reconsiders the ideas presented in light of connections with other mathematical ideas, and includes questions for readers' reflection.

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