

NUMBER & MEASUREMENT • GEOMETRY

STATISTICS & PROBABILITY

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Focus in High School Mathematics

Reasoning and Sense Making



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Focus in High School Mathematics: Reasoning and Sense Making

A dramatic culture change needs to take place in the high school mathematics curriculum, and this change will require engaging stakeholders at all levels.

Addressing the direction of high school mathematics in the twenty-first century, *Focus in High School Mathematics* builds on the ideas of NCTM's *Principles and Standards for School Mathematics* (2000) and focuses on how high school mathematics can better prepare students for future success.

Reasoning and sense making are at the heart of the high school curriculum. Discover the components of mathematical reasoning and sense making in grades 9–12, including—

- reasoning habits that instruction should develop across the curriculum,
- key elements for reasoning in five content strands, and
- examples that illustrate the roles of teachers and students in the classroom.

Focus in High School Mathematics aims to bring together teachers, teacher educators, administrators, supervisors, curriculum specialists, mathematicians, and statisticians to reshape the conversation about high school mathematics and shift the curriculum toward an emphasis on reasoning and sense making for all students.

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