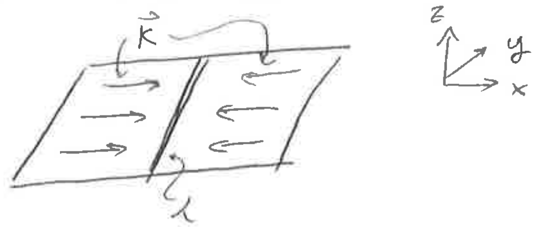
PHYS 542 Homework 1

1 Charge accumulation on a plane A time independent surface current with density $\pm\mathbf{K}$ flows in the $x-y$ plane inwards towards the line at $x=0$. As a result, a line charge accumulates at $x=0$ at a constant rate $d\lambda/dt$.

(a) Find the displacement current everywhere (express this in terms of the magnitude of the surface current K).

(b) Find the magnetic field everywhere above and below the plane. Check that the boundary conditions at $z=0$ and $x=0$ are sensible.



2 Dipole down the tube A small magnet (weight w) falls under gravity down the center of an infinitely long, vertical and conducting tube of radius a, wall thickness $t << R$ and conducitivity σ. Let the tube be concentric with the z-axis and model the magnet as a point dipole with moment $\mathbf{m} = m\hat{\mathbf{z}}$. We can find the terminal velocity of the magnet by balancing the weight against the magnetic drag force associated with ohmic loss in the walls of the tube.

(a) At the moment the magnet passes through $z=z_0$, show that the magnetic flux produced by $\mathbf{m}$ through a ring of radius a and height z' is

$$\Phi_B = \frac{\mu_0 m}{2}\frac{a^2}{r_0^3}$$

where $r_0^2 = a^2 + (z_0 - z')^2$

(b) When the speed v of the dipole is small, argue that the Farady EMF induced in the ring is:

$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t} = v\frac{\partial \Phi_B}{\partial z'}$$

(c) Show that the current induced in the thin slice of the tube which includes the ring is

$$dI = \frac{3\mu_0 m a v \sigma t}{4\pi}\frac{z_0 - z'}{r_0^5}dz'$$

(d) Compute the magnetic drag force $\mathbf{F}$ on $\mathbf{m}$ by equating the rate at which the force does work to the power dissipated in the walls by Joule heating.

$$\mathbf{F}\cdot\mathbf{v} = \int \mathcal{E}\, dI$$

(e) Find the terminal velocity of the magnet.