PHYS 542 Homework 2

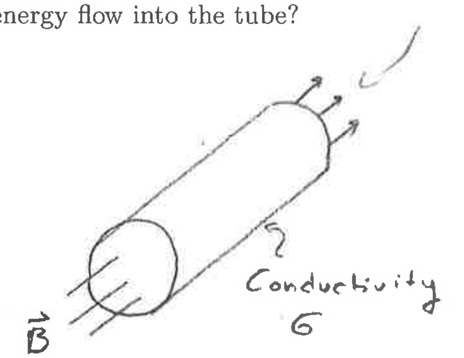
1. Azimuthal Eddy Currents in a Wire A longitudinal AC magnetic field $\mathbf{B}(t) = \hat{\mathbf{z}} B_0 \sin\omega t$ is driven through the interior of an ohmic tube with conductivity σ, length L and radius $R << L$.

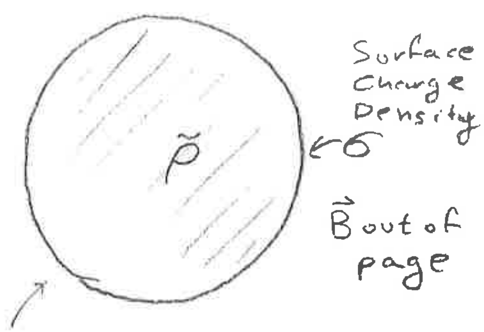
(a) Find the low frequency eddy current density inside the tube $\mathbf{j}_0$, assuming the quasi-magnetostatic approximation (i.e. ignoring the magnetic fields produced by self-induction).

(b) At this same level of approximation, compute the Poynting Flux through the surface $\rho = R$ of the tube. At this level of approximation, what is the time-average energy flow into the tube?

(c) Find the correction to the eddy current density produced by self-induction $\mathbf{j}_1$ (to lowest order in ω).

(d) Compute the Poynting Flux through the surface $\rho = R$ of this tube due to this correction in the eddy current density. At this level of approximation, what is the time-average energy flow into the tube?





2 Transformation of Angular Momentum The figure shows a cutaway view of an infinite cylindrical solenoid with radius R that creates a magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ inside itself. An infinitely long cylinder of insulating material with $a < R$, permeability μ_0 and permittivity ϵ_0 sits inside (and coaxial with) the solenoid. The cylinder is filled uniformly with charge density $\bar{\rho}$. A uniform charge σ makes the cylinder electrically neutral.

(a) Find the total electromagnetic angular momentum (per unit length) of this system.

(b) Compute the instantaneous torque (per unit length) which acts on the cylinder as the magnetic field is reduced to zero with an arbitrary time dependence $B(t)$ (i.e. express the torque in terms of $\partial B/\partial t$).

(c) Show that the final mechanical angular momentum of the cylinder is equal to the initial angular momentum calculated in part (a).