PHYS 542: Homework 3

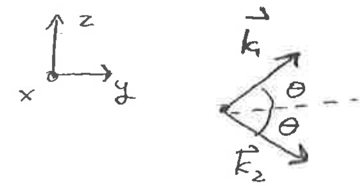
1. Superpositions of Plane Waves

(a) Let $\mathbf{E} = E_0 \sin(kz - \omega t)\hat{\mathbf{x}} + E_0 \sin(kz + \omega t)\hat{\mathbf{x}}$. Write $\mathbf{E}(z,t)$ in the form of a standing wave and find the associated magnetic field $\mathbf{B}(z,t)$.

(b) For the fields in part (a), find the instantaneous and time-averaged electric and magnetic field energy densities. Find also the instantaneous and time averaged energy current density.

(c) Let $\mathbf{E} = E_0 \sin(kz - \omega t)\hat{\mathbf{x}} + E_0 \cos(kz + \omega t)\hat{\mathbf{y}}$. Determine the polarization of this field at $z = 0, \lambda/8, \lambda/4, 3\lambda/8$ and $\lambda/2$

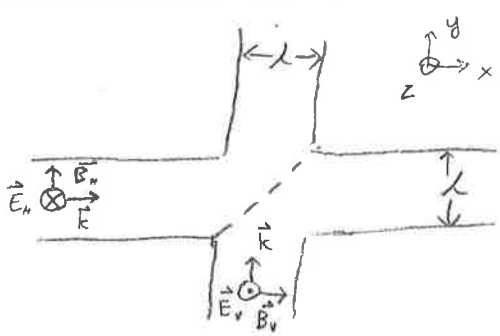
(d) Show that the field in part (c) can be written as the superposition of an LHC standing wave and a RHC standing wave.



(e) Consider the superposition of two equal-amplitude, time-harmonic, x-polarized plane waves. Their wave vectors satisfy $|\mathbf{k_1}| = |\mathbf{k_2}|$ and lie in the $y - z$ plane as shown here. Is the sum of these two plane waves itself a plane wave? is it transverse?

2. When Interference Behaves like Reflection

The diagram shows two electromagnetic beams intersecting at right angles $(\mathbf{E_H}, \mathbf{B_H})$ propagates in the $+x$ direction. $(\mathbf{E_V}, \mathbf{B_V})$ propagates in the $+y$ direction. For simplicity, each beam is taken as a pure plane wave (with $\omega = ck = 2\pi c/\lambda$) cut off transversely so its cross section is a perfect square of area λ^2.



$$\mathbf{E_H} = -E_0 e^{i(kx-\omega t)}\hat{\mathbf{z}}, |y| \le \lambda/2, |z| < \lambda/2$$
$$c\mathbf{B_H} = +E_0 e^{i(kx-\omega t)}\hat{\mathbf{y}}, |y| \le \lambda/2, |z| < \lambda/2$$
$$\mathbf{E_V} = +E_0 e^{i(ky-\omega t)}\hat{\mathbf{z}}, |x| \le \lambda/2, |z| < \lambda/2$$
$$c\mathbf{B_V} = +E_0 e^{i(ky-\omega t)}\hat{\mathbf{x}}, |x| \le \lambda/2, |z| < \lambda/2$$

The beams overlap in a cube of volume λ^3 centered on the origin where the total fields are $\mathbf{E} = \mathbf{E_H} + \mathbf{E_V}$ and $\mathbf{B} = \mathbf{B_H} + \mathbf{B_V}$

(a) Calculate the time-averaged energy density$\langle u_{EM}(\mathbf{r})\rangle$ for the horizontal beam alone (H), for the vertical beam alone (V), and for the total field in the overlap region. Show that the latter takes its minimum value on the plane $x = y$ shown as a dashed line in the diagram. Compute $\mathbf{E}$ and $\mathbf{B}$ in this plane.

(b) Calculate the time-averaged Poynting Vector $\langle \mathbf{S}(\mathbf{r})\rangle$ for the H Beam, the V Beam and the total field. Make a careful sketch of $\langle \mathbf{S}(x,y)\rangle$ everywhere the fields are defined.

(c) Explain why the behavior of both $\mathbf{E}$ and $\mathbf{B}$ in the vicinity of the $x = y$ plane is exactly what you would expect in that plane were a perfect conductor. This shows that the interfering beams behave as if they had specularly reflected from each other.