PHYS 542 Homework 5

1. Airy's Problem Revisited Airy's problem is the transmission of a monochromatic plane wave through a transparent film (ϵ, μ) of thickness d. The text solved this problem by summing an infinite number of single-interface Fresnel reflections and transmissions. For this problem specialize to the case of *normal incidence* and use the matching conditions and a five-wave analysis (a forwards going wave in the $z > d$ vacuum, a forward-going and a backward-going wave in the $z < 0$ vacuum and a forward-going and a backward-going wave in the film to show that the fraction of the incident power transmitted into the vacuum through the $z = d$ surface of the film is:

$$|T|^2 = \left| \frac{4ZZ_0}{(Z+Z_0)^2 - (Z-Z_0)^2 e^{2ikd}} \right|^2$$

where Z_0 and Z are the wave impedances in the vacuum and in the film.

2. Refraction into a Good Conductor Consider plane wave refraction from a non-conducting medium (ϵ, μ) into a conducting medium (ϵ, μ, σ). Ohmic loss requires that the refractive wave vector $\mathbf{k}_2$ be complex. The figure below shows a proposed refraction geometry where $\mathbf{k}_2 = \mathbf{q} + i\boldsymbol{\kappa}$

(a) Explain why $\boldsymbol{\kappa}$ points in the $+z$ direction and why the angle of incidence still equals the angle of reflection outside the conductor.

(b) Derive the generalization of Snell's law for this problem. Use appropriate dispersion relations to re-write the appropriate wavevector magnitudes in terms of σ, ω, ϵ, μ and the relevant angles. You do not have to simplify this relationship to any specific form.