PHYS 542 Homework 7

1. Limits on the Photon Mass If the photon had a mass M, the dispersion relation for electromagnetic waves in vacuum would be

$$\omega^2 = c^2k^2 + (Mc^2/\hbar)^2$$

A limit on $Mc^2 << \hbar\omega \simeq \hbar ck$ can be determined by measuring the difference in arrival times of the highest- and lowest-frequency components of a wave packet received from an astrophysical source that emits EM radiation in bursts. Estimate δt from the proposed dispersion relation. Should one collect data from a radio source or from a γ-ray source to obtain the lowest limit on M? Why?

2 Negative Group Velocity An infinite slab of material with index of refraction $n(\omega)$ and group velocity $v_g < 0$ occupies a space $0 < z < a$. The rest of space is a vacuum.

(a) Consider a plane wave with electric field $\mathbf{E} = \hat{\mathbf{x}}E_0e^{i\omega(z/c-t)}$ incident on the slab from $z < 0$. Use $n(\omega)$ to write formulae for the wave in regions $0 < z < a$ and $z > a$. Assume $n(\omega)$ is not far from unity so reflection from the slab surfaces can be ignored. Write these expressions in a way that clearly distinguishes the frequency-dependent terms.

(b) Make the group velocity approximation $\omega n(\omega) \simeq \omega_0 n(\omega) + (c/v_g)(\omega - \omega_0)$ and write formulae for the field $E(z,t)$ in all three regions.

(c) Let $E(z < 0, t) = f(z/c - t) = \int d\omega \hat{E}(z,\omega)e^{-i\omega t}$ be a wave packet incident on the slab . Find expressions for $E(z,t)$ for $0 < z < a$ and $z > a$ with the form $e^{i\Phi}f(x)$ using the Fourier components derived in part (b), where x is a function of z and t that might be different for the three regions.

(d) Choose $f(x) = E_0\delta(x)$, write down the field and verify (1) that the packet travels "backwards" through the slab and (2) that the packet exits the slab before it enters.