Problem Solving Stages

Lesson VIII: Problem Solving
module 37

University of Idaho

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Problem Solving.37.

Taxonomy of problems

- Goals and operators
  - When trying to solve a problem, the problem solver is trying to achieve a particular goal
  - Operators refer to all possible actions / paths a problem solver can take at each point in time

- Well-structured problems
  - Clear path to a solution
  - Goals and operators are known

- Ill-structured problems
  - Unclear path to a solution
  - Neither the goal, nor the operators, sometimes even the problem itself is clear

The range of problems

- Well-structured problems
  - \( 4 + 5 = ? \)
  - The ratio of blue socks to red socks is 5:1. How many socks do you have to pick blindfolded until you have at least one pair of the same color.
  - Cannibals and missionaries

- Ill-structured problems
  - Connect the following three points with four straight lines without moving the pen from the paper.
  - \$10 challenge: can you do it with 3 straight lines (again, don’t move the pen from the paper!)
Problem solving - an overview

- How do humans solve problems?
- What are common problem-solving problems?
  - Stages in the problems-solving process
  - Problem solving strategies / heuristics
  - What is insight? Incubation?
  - Problems in problem solving:
    - Mental set, functional fixedness, transfer
    - Analogies, problem structures
  - What makes an expert problem solver?

Stages of the problems solving process

- Stage model according to Sternberg
  - Problem identification
  - Problem definition and representation
  - Strategy formulation / implementation
  - Organization / interpretation of information
  - Resource allocation
  - Monitoring
  - Evaluation

Problem definition and representation

- The basic problem
  - The ease of a solution often depends on
  - ... how the problem is phrased (its surface structure)
  - ... and what underlying assumptions are “imported”
- Applied problems
  - Most real problems are ill-defined
  - Sometime not even the outcome measure can be agreed upon (e.g., policy making)
  - Examples: how can we improve public education?
A sample problem

- Identify the rule behind each sequence. What is the next number?
- What is the rule behind each sequence?
  1 2 3 4 5 6 7 …?
  1 8 2 7 3 6 4 …?
  8 5 4 9 1 7 6 …?

Problem structure

- Deep structure vs. surface structure of a problem
  - The deep structure of a problem refers to the abstract form of the problem
  - The surface structure of a problem refers to the context and in which the problem is embedded
- Isomorphic problems
  - … are two problems which only differ in their surface structure, but are based on an identical deep structures
  - Example: An accountant can apply his business knowledge to all kinds of enterprises

Isomorphic problem structure

- Banana vs. milk example
  - GOAL: you want to cut off a quarter of a banana
  - You first cut the banana in half
  - Then you cut one of the two halves
  - GOAL: you want to get a quarter of a gallon of milk and you only have two large, equal containers of which you don't know the volume
  - First step: you fill each of the containers so that they hold the same amount
  - Second step: you dump the content of one and distribute the remaining content between equally between the two containers
- Accounting example
  - An accountant can apply his business knowledge to all kinds of enterprises
Most well-defined problems can be represented in a problem space:
- Define the current state
- Define all possible transitions from one state to another state using the possible operators
- Define the goal state

Problem solving then consists of...
- A path from the current state to the goal state
- The transitions indicate the operators to be used
- Computers can use algorithms to find possible paths by trying a huge number of paths

Humans often rely on heuristics
- heuristic: strategy that often works effectively

Analysis and hierarchical problem solving
- Breaking the problem up into sub-problems
- Solve series of sub-problems until done

Heuristics
- Means-ends analysis: Reduce distance between current state and goal state
- Working forward, backward
- Generate and evaluate

Problem solving by analogy
- Try to find an analogous problem (isomorphic problem structure) that has been solved
- Transform the solution to new problem

Which piece of information is relevant?
- Depending on the state of the problem solving process different parts of the given information are relevant or have to be sought out

Reinterpretation of information
- Value / interpretation of information can change:
- Example: the series problem earlier in this module
- The woman who married 20 men (textbook)
Resource allocation and monitoring

- Strategic decisions - metacognition
  - How to allocate time? Energy? Money?
  - Experts emphasize planning stage
  - Novices embark on "local" planning and execution
- Monitoring
  - State of the problem solving progress
  - Re-evaluation of problem solving approach
  - Evaluation of distance from goal
  - Estimate of problem completion

Evaluation

- Is this solution a valid / workable solution?
  - Checking solution against constraints
  - Reorganization of problem solving steps
  - Cleaning up
- Feeling accompanying insight - "ah-ha" experience
  - Insight: distinctive and sometimes seemingly sudden understanding of a problem or strategy to solve the problem. Often reconceptualizing a problem in a new way.
  - Pieces "fall into place" - Evaluation feels redundant.