Stat/Math 452 (EO), Fall 2023
Assignment 2
Due (via email to instructor): target date approx Nov 29

Suppose $n$ independent, identically distributed observations are drawn from an exponential($\lambda$) distribution, with pdf given by

$$f(x \mid \lambda) = \lambda e^{-\lambda x}, \ 0 < x < \infty.$$

The data are $x_1$, $x_2$, ..., $x_n$.

(1) Construct a likelihood ratio hypothesis test of H$_0$: $\lambda = \lambda_0$ vs H$_1$: $\lambda = \lambda_1$ (where $\lambda_1$ and $\lambda_2$ are known constants, with $\lambda_0 < \lambda_1$), where the critical value is taken to be a constant $c$.

(2) Show how the likelihood ratio test constructed in (1) reduces to comparing $\overline{x}$ to a critical value (denoted, say, by $\overline{x}_c$).

(3) Use what you know about the sample mean to specify a method of picking $\overline{x}_c$ in order to construct a size $\alpha$ test under the Neyman-Pearson framework.

(4) Expand your test into a test of H$_0$: $\lambda = \lambda_0$ vs H$_1$: $\lambda > \lambda_0$. Argue whether or not the resulting test is a uniformly most powerful test.

(5) Show how one would obtain a $p$ value for the test in (4). As well, show how one would calculate power for the test.

(6) Construct a generalized likelihood ratio test of H$_0$: $\lambda = \lambda_0$ vs H$_1$: $\lambda \neq \lambda_0$. Use Wilk's theorem to obtain an approximate distribution of the test statistic under H$_0$, and use the distribution to specify how to calculate a $p$ value for the test.

(7) Simulate 1000 samples of size $n = 5$ from model H$_0$, using $\lambda_0 = 0.1$. For each sample, calculate the value of the test statistic proposed in (6). Compare the 1000 test statistic values to the approximate distribution obtained in (6) with a probability plot (constructed for that distribution). You can use any computational software.

(8) Repeat (7) using a sample size of $n = 10$.

(9) Repeat (7) using a sample size of $n = 15$.

Hand in via email (pdf or MS Word file):
I. Cover sheet, with name and typed paragraph describing the simulation results.
II. Derivations, neatly hand written, or typed (LaTeX, etc.). Must be in pdf or MS Word file (pdf preferred).
III. Three probability plots, as separate figures or as separate panels in one figure.