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Similarity Solutions of Natural Convection With Internal Heat Generation

J. C. Crepeau[1], and R. Clarksean[2]

Nomenclature

c_p = specific heat at constant pressure
$f(\eta)$ = function defined in Eq. (5)
$f'(\eta)$ = nondimensional velocity
g = gravity constant
Gr_x = Grashof number, $g\beta(T_s - T_\infty)x^3/\nu^2$
k = thermal conductivity
L = length
Nu = Nusselt number
Pr = Prandtl number
q''' = internal heat generation per unit volume
T = temperature
T_s = surface temperature
T_∞ = ambient temperature
u, v = velocity in the x, y direction
x, y = Cartesian coordinates
β = volumetric coefficient of thermal expansion
η = similarity variable defined in Eq. (4)
ν = kinematic viscosity
ρ = density
ψ = similarity variable defined in Eq. (5)
$\theta(\eta)$ = nondimensional temperature, $(T - T_\infty)/(T_s - T_\infty)$

1 Introduction

A large number of physical phenomena involve natural convection driven by internal heat generation. An example includes convection in the earth's mantle (McKenzie, et al., 1974), but perhaps the most widespread application is in the field of nuclear energy. In nuclear reactor cores (Smith and Hammitt, 1966) and in postaccident heat removal (Baker et al., 1976), natural convection driven by internal heat generation plays an important role in the overall heat transfer. Natural convection with internal heat generation also applies to fire and combustion modeling (Delichatsios, 1988), the development of a metal waste form from spent nuclear fuel (Westphal, 1994), and for the storage of spent nuclear fuel.

Theoretical, numerical, and experimental analyses have been completed on natural convection with internal heat generation. Typical examples include Tritton and Zarraga (1967), Roberts (1967), and Jahn and Reineke (1974). In view of the amount of work done on natural convection with internal heat generation, it is worthwhile to determine if similarity solutions exist for this system. Similarity solutions of natural convection along a vertical isothermal plate have been shown by Ostrach (1953). The similarity solution results are useful in understanding the interaction of the flow field and temperature field. The trends exhibited can allow the scientist or engineer to determine over what range of internal heat generation rates and Prandtl numbers the addition of internal heat generation needs to be considered.

A similarity solution for a fluid with an exponentially decaying heat generation term and a constant temperature vertical plate is developed. An exponential form is used for the internal energy generation term. All of the numerical solutions were obtained through the use of *Mathematica* (Wolfram, 1991). The procedure used to solve the resultant differential equations was validated by obtaining the solutions for a constant temperature vertical plate without internal heat generation as shown in Fig. 1.

2 Problem Derivation

Consider a vertical plate in a semi-infinite quiescent fluid. The temperature of the plate is held at constant value T_s, and the fluid has an internal volumetric heat generation, q'''. By taking x to be along the plate in the vertical direction, and y perpendicular to the plate, the governing equations (continuity, momentum, and energy) for the Boussinesq approximation within the boundary layer are given by,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + q'''. \tag{3}$$

Similarity variables of the form (Jaluria, 1980),

$$\eta(x, y) = \frac{y}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4} \tag{4}$$

$$\psi(x, y) = 4\nu f(\eta)\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4} \tag{5}$$

are introduced into Eqs. 1–3. In order for a similarity condition to exist, the volumetric heat generation must be of the form

$$q''' = \frac{k(T_s - T_\infty)}{x^2}\left(\frac{\mathrm{Gr}_x}{4}\right)e^{-\eta}.$$

By inserting the similarity variables and the nondimensional temperature into the governing equations, the following similarity equations result:

[1] Department of Mechanical Engineering, University of Idaho, P.O. Box 50778, Idaho Falls, ID 83405. Assoc. Mem. ASME.

[2] Clarksean and Associates, Ottertail, MN 56571. Assoc. Mem. ASME.

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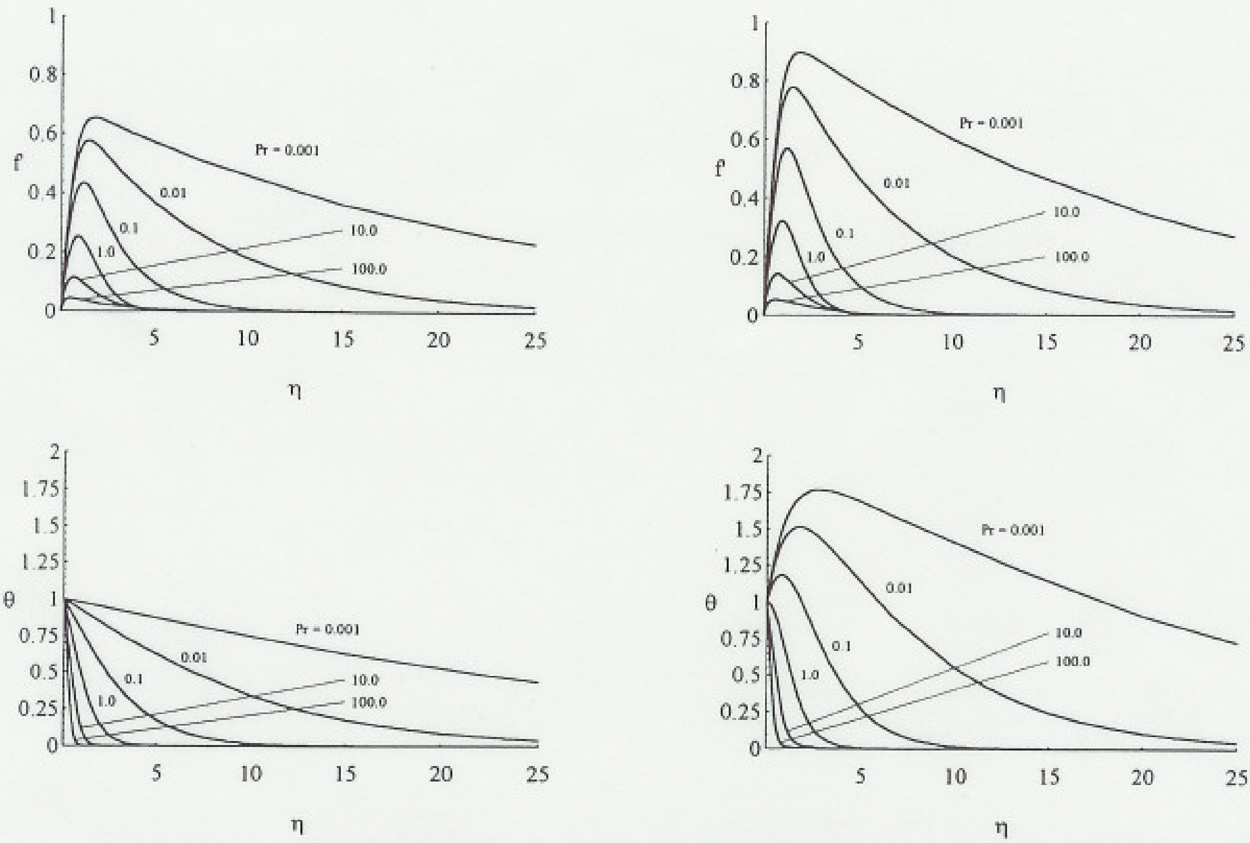

Fig. 1 Solutions of Eqs. 6 and 7 for a constant temperature plate and various Prandtl numbers. Top curves represent velocity profiles, bottom curves temperature profiles. Curves on the left are for fluids without internal heat generation; curves on the right are for fluids with internal heat generation.

$$f''' + 3ff'' - 2f'^2 + \theta(\eta) = 0 \tag{6}$$

$$\theta'' + 3\,\mathrm{Pr}\, f\theta' + e^{-\eta} = 0 \tag{7}$$

with boundary conditions,

$$f(0) = f'(0) = f'(\infty) = 0$$

$$\theta(0) = 1,\ \theta(\infty) = 0.$$

The exponentially decaying heat generation model can be used in mixtures where a radioactive material is surrounded by inert alloys and has been used to model electromagnetic heating of materials (Şahin, 1992).

3 Results and Discussion

The solutions to Eqs. 6 and 7 over a range of Prandtl numbers are given in Fig. 1. The plots on the left hand side are for a fluid without the exponentially decaying heat generation term, while those on the right include $e^{-\eta}$. The nondimensional velocity f' plots are on the top, and the nondimensional temperature θ curves are shown on the bottom. The effect of the internal heat generation is especially pronounced in the low Prandtl number profiles.

It is interesting to note in Fig. 1 that $d\theta/d\eta|_{\eta=0}$ is zero for a Pr of approximately one. At this point, there is no heat transfer to or from the fluid. The flow develops as a result of the internal energy generation and is not impacted by the constant temperature vertical plate. For $\mathrm{Pr} < 1$, $\theta_{\max} > 1$, which indicates the importance of the thermal properties.

The similarity velocity f' is greater when internal energy generation exists. This is logical because the internal energy generation results in an increase in the buoyancy forces which will induce more flow along the plate. In Fig. 1 the location of the maximum velocity occurs at roughly the same value of η for $\mathrm{Pr} \leq 1$. For $\mathrm{Pr} \geq 1$, the location of the maximum velocity occurs at a larger distance from the plate, indicating the influence of the increased viscosity.

From the figure, the average Nusselt number along a plate of length L can be determined (Incropera and DeWitt, 1990) by,

$$\overline{\mathrm{Nu}}_L = -\frac{4}{3}\left(\frac{\mathrm{Gr}_L}{4}\right)^{1/4} \frac{d\theta}{d\eta}\bigg|_{\eta=0}$$

where the gradients are shown in Table 1. In addition to heat transfer applications, determination of the Nusselt number is important in solidification processes since convection is the dominant heat transfer mode by an order of magnitude over conduction (Yao, 1984). Not only does an enhanced Nusselt number cause tremendous changes in the solidification rate (Yao and Prusa, 1989), but it also affects the shape of the solid-liquid interface and the alloy structure of the solid (Viskanta, 1988).

4 Conclusions

Similarity solutions have been developed to analyze the effect of an exponential form for internal heat generation for a constant temperature vertical plate. A range of Pr has been examined. As expected, the presence of internal energy generation leads to increased flow, and in some cases, temperatures that exceed the wall temperature, especially for fluids with $\mathrm{Pr} < 1.0$. The effect of internal heat generation is important in several applications that include reactor safety analyses, metal waste form development for spent nuclear fuel, fire and combustion studies, and the storage of radioactive materials.

Table 1 Computed values of $d\theta/d\eta|_{\eta=0}$ for the similarity equations for a constant temperature plate (Equations 6 and 7) with and without internal heat generation (q''')

Pr	$d\theta/d\eta\|_{\eta=0}$, constant temperature plate	
	without q'''	with q'''
0.001	-0.02640	0.9392
0.01	-0.08059	0.8236
0.1	-0.2302	0.5425
1.0	-0.5671	0.005786
10.0	-1.169	-0.7963
100.0	-2.191	-1.979

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Benchmark Solutions of Radiative Heat Transfer Within Nonhomogeneous Participating Media Using the Monte Carlo and YIX Method

Pei-feng Hsu,[1] J. T. Farmer[2]

Introduction

Solutions of radiative heat transfer in fire and combustion systems have been an important and active research subject in recent years. Most of the applications encountered in actual systems contain nonhomogeneous participating media. It is important that the flames and combustion systems are treated as such due to the non-uniform distributions of temperature and absorbing gaseous species, and participating particle concentrations. While several methods (Siegel and Howell, 1992) have treated some benchmarks successfully, the lack of the direct comparisons of any nonhomogeneous medium benchmark solution using different methods motivated the current study. In this study, we use two different methods (one stochastic—the Monte Carlo, and one deterministic—the YIX) to solve radiative transfer within the three-dimensional, nonhomogeneous, absorbing, emitting, scattering media and compare the solution differences between the two methods. The results are tabulated for future comparisons with other solution techniques.

In the first Symposium on Solution Methods for Radiative Heat Transfer in Participating Media at the 1992 National Heat Transfer Conference (Tong and Skocypec, 1992), the YIX (Hsu et al., 1993) and the Monte Carlo (M.C.) methods (Farmer and Howell, 1994) were shown to be able to solve complicated radiative heat transfer problems that have nonhomogeneous and nongray participating media. It was later shown that the differences between the two methods are within five percent (Hsu, 1994). Our experience leads us to suspect that the difference in spectral integration techniques is major cause of discrepancy. One of the purposes of this study is to confirm the agreement of the two methods for a series of gray nonhomogeneous media with and without anisotropic scattering as a first step.

Benchmark Problems

The geometry of the problem is a unit cube with black walls. Several different cases are treated. All cases have similarly shaped optical thickness (τ = extinction coefficient times the side length) distribution given by Eq. (1), i.e.,

[1] Mechanical & Aerospace Engineering Programs, Florida Institute of Technology, Melbourne, FL

[2] Thermal and Life Support Division, NASA Marshall Space Flight Center, Huntsville, AL

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