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Spectral Entropy Measurements of Coherent Structures in an Evolving Shear Layer

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Abstract

We employ a spectral entropy analysis to show that coherent vortex structures are more ordered than the surrounding fluid in which they exist. By excising these structures from the time-series data, one can compare their relative order with the laminar-transition state fluid that encompasses the bursts. The data show that as the vortex structures progress downstream, they dissipate into the flow. This dissipation is marked by an increase in the spectral entropy.

To gain an understanding of the spectral entropy behavior around the critical layer, we took samples along a free shear layer in an internal cavity. Measurement locations ranged from the steep shear region near the forward restrictor to the fully developed flow region. The data taken along the critical layer in the steep shear region reflect relatively low values of the spectral entropy with higher values existing in a small region above the critical layer. This behavior suggests that the experimentally observed vortex structures are created along the critical layer. The downstream increase in spectral entropy may indicate the dissipation of structure, or a transformation into a structure with a lower valued energy.

In addition, we show that the flow field may be divided into linear non-equilibrium and nonlinear non-equilibrium (far from equilibrium) branches. The linear non-equilibrium branch contains no structure and the spectral entropy reaches a maximum. Self-organization takes place as the flow moves onto the nonlinear non-equilibrium branch. On this branch, the spectral entropy decreases.

Introduction

In a previous work, Crepeau and Isaacson [1] demonstrated how the spectral entropy serves as an order parameter and decreases during a self-organizing process. They performed their numerical computations on three separate dynamical systems: the logistic equation, the Lorenz equations, and a modified form of the Townsend equation which models free shear layers. The authors' results show that self-organization occurs as the system bifurcates onto the non-equilibrium thermodynamic branch.

Both Prigogine [2] and Haken [3] have studied the transition from the linear nonequilibrium branch to the far from equilibrium region and have indicated that this transition can lead to self-organization. The S-theorem of Klimontovich [4] confirms these findings for turbulent flows. Ebeling et al. [5] explicitly show that laminar and turbulent structures, both of which are formed far from equilibrium, have a lower entropy than the equilibrium state when the total energy is held constant. We show in this paper how the spectral entropy emulates these results. The spectral entropy is simple to compute and the techniques can be applied to experimental data.

In this work, we analyze experimental data using spectral entropy techniques on coherent structures in an internal shear layer, far from any wall disturbances, and compare their characteristics with the surrounding flow. In addition, we show how the spectral entropy can locate regions of self-organization from data taken within the transition flow field along the shear layer.

The spectral entropy, as proposed by Powell and Percival [6], gives a measure of the broadbandedness of a power spectrum. They define the spectral entropy as,

$$S = -\sum_k P_k \log P_k \tag{1}$$

where,

$$P_k = \frac{|f_k|^2}{\sum_{k'} |f_{k'}|^2}. \tag{2}$$

Here, $|f_k|^2$ represents the value of the power spectrum at a particular wavenumber. A power spectrum with one or two dominant frequencies possesses a relatively low spectral entropy, and a broadbanded spectrum has a higher value.

A coherent structure contains a few dominant frequencies and therefore should have a lower spectral entropy than the background flow. The background flow, which is made up of a wide range of frequencies, contains no structure and therefore has a higher spectral entropy. A signal with totally random fluctuations is comprised of all frequencies, each appearing with equal probability. The maximum spectral entropy, S_{max}, becomes $\log(N/2)$, where N is the number of data points in the measurement. Intuition demands that an "ordered" structure has a lower entropy than the surrounding, structureless, environment.

In Sections 1 and 2, we demonstrate how the spectral entropy of coherent structures in an internal free shear layer compares with that of the surrounding fluid. We track the evolution of these structures as they progress downstream and dissipate into the fluid field. In addition, the results compare the spectral entropy measured along the critical layer, with the spectral entropy values about one millimeter above the layer. The critical layer is defined as the locus of inflection points of the velocity profiles. We have shown [7] that for the flow cavity used in this work, the velocity profiles are modeled by modified (Stuart [8]) $\tanh(y)$ curves. Section 3 describes how the spectral entropy locates regions where self-organization occurs. The algorithms given by equations 1 und 2 serve as an indicator to the origin of the creation of structure in the flow.

1. Spectral entropy measurements of individual structures

Isaacson [9] has reported the presence of spiral vortices in the steep shear region of an internal shear layer far away from wall disturbances. In that paper he describes the experimental apparatus which is used in these experiments, including the flow cavity. The cavity was originally built to study vortex-driven acoustic oscillations within segmented solid propellant rocket motors.

By placing an *X*-film anemometer along the critical layer, we can record the velocity characteristics of the coherent structures. The top portion of Figure 1 gives a typical velocity *vs.* time trace. Here, the anemometer is placed three millimeters behind the forward restrictor and along the critical layer, 21.1 mm below the centerline of the cavity. One can observe three well-defined structures which exist in the background laminar flow. We attribute these unsteady bursts to the vortices reported by Isaacson [9].

The spectral entropy, as described by Equation 1, now represents an order parameter and can be computed for each structure in the flow. After computing the spectral entropy for individual structures, we can calculate it for the surrounding flow regions. This permits us to compare the amount of structure which is contained in different portions of the flow field.

The bottom curve in Figure 1 shows how the spectral entropy varies for certain samples within the time-series. Each sample corresponds to either a coherent structure or the surrounding flow. Note that the order parameter decreases in the regions where the structures appear, and increases in the surrounding flow regions. The decrease in the spectral entropy indicates that the coherent structures exist in a more ordered state than that of the surrounding flow. Similar results on time-series data have been reported [10] previously.

While the results present the spectral entropy behavior of individual structures in the flow, one needs to consider how the spectral entropy, on average, varies for the coherent structures at different locations in the flow field. The next section examines the average spectral entropy for many bursts and surrounding flow regions along various locations around the critical layer.

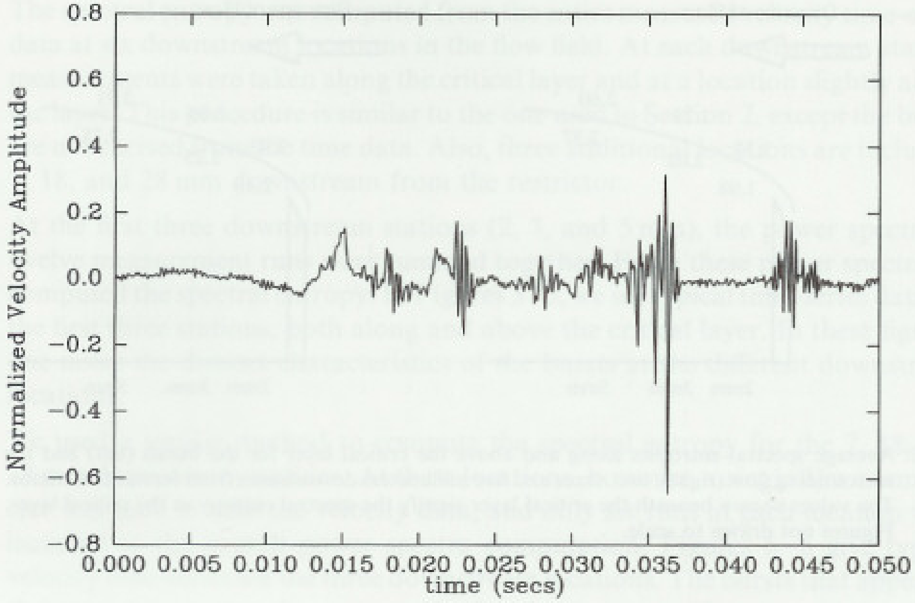
2. Average spectral entropy of coherent structures and surrounding flow

Through understanding the average spectral entropy behavior of the bursts and the surrounding flow throughout the entire field, one gains important insight into the overall structure of the flow. In this section, we will concentrate on the dissipation of the vortex structures in the steep shear region of the flow, two, three, and five millimeters downstream of the forward restrictor.

At each downstream location, the probe is placed along the critical layer where ten measurements are made. Before changing to the next downstream location, the probe is moved about one millimeter vertically where ten more measurements are taken. Each measurement consists of 512 data points, taken in 0.05 seconds. For each time-series measurement, the burst regions are excised from the surrounding flow, similar to the procedure described in Section 1. Instead of computing the arithmetic average of the spectral entropy for the bursts, the power spectra for all bursts are summed together. From the summed power spectra, we then compute the spectral entropy. A similar analysis is performed on the surrounding flow regions. By computing the spectral entropy in this manner, we follow the Boltzmann concept of entropy. He devised the entropy as a measure of the number of available states in the system. Adding the power spectra together enables us to observe the magnitude and availability of each state. Table 1 summarizes the results.

At each downstream station we observe that the spectral entropy is significantly lower along the critical layer than at a location slightly above it. This implies that the critical layer is the processing region which creates the observed vortex structures. One may observe these trends in Figure 2. The spectral entropy behavior shown in Figure 2 also reveals the dissipation within the structures. As the flow progresses downstream, the spectral entropy increases both along the critical layer, and in the region about one millimeter above the layer. This increase in the spectral entropy indicates the breakdown of the coherent structures into structures with lower valued energy.

The spectral entropy of the surrounding flow is significantly higher than the bursts. This confirms the conclusion in Section 1 that the bursts are more ordered than the surrounding flow. As with the bursts, the spectral entropy increases as the fluid moves downstream. The structures, however dissipate into the flow and affect the entire field.



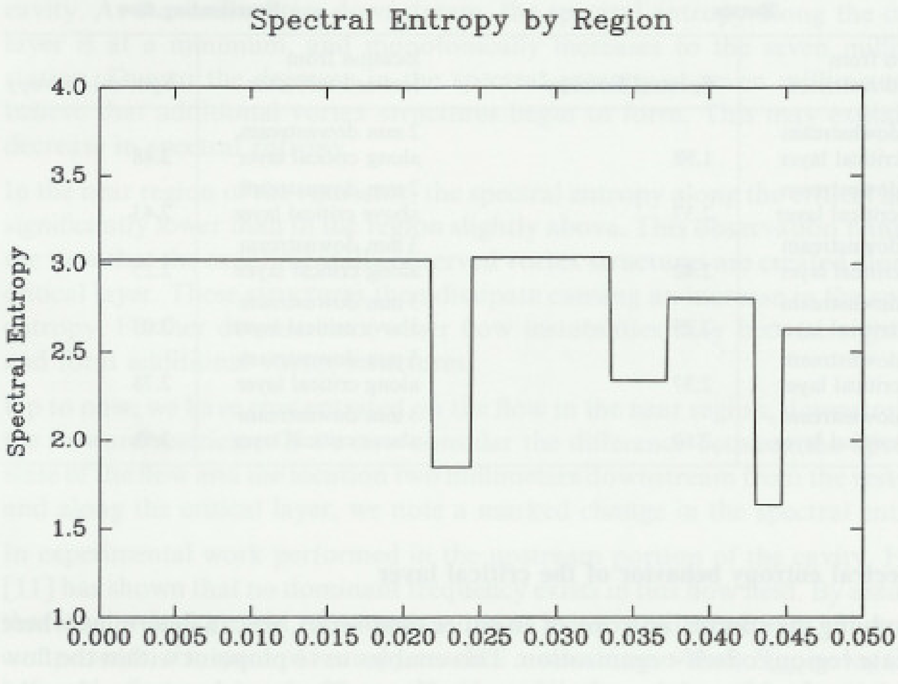


Fig. 1: Top: Velocity fluctuations *vs.* time where the probe is 3 mm downstream from the forward restrictor and slightly (~1 mm) above the critical layer. Bottom: Spectral entropy by region of the coherent structures and the surrounding flow.

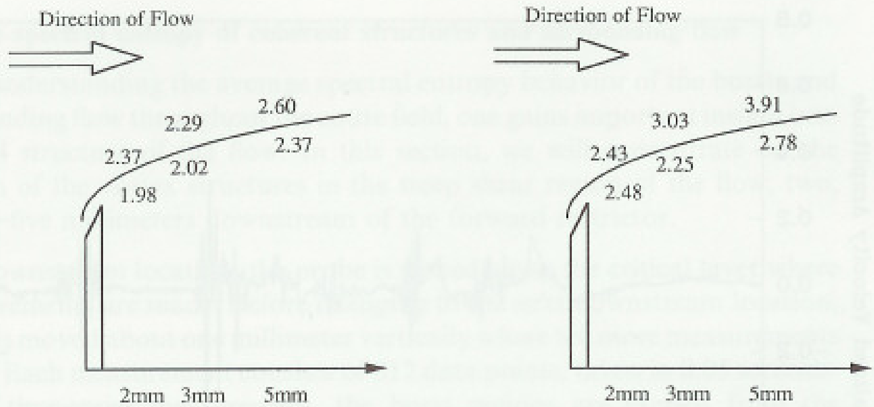


Fig. 2: Average spectral entropies along and above the critical layer for the bursts (left) and the surrounding flow (right), two, three and five millimeters downstream from forward restrictor. The values shown beneath the critical layer signify the spectral entropy at the critical layer. Figures not drawn to scale.

Tab. 1: The variation of the spectral entropy of both the bursts and the surrounding flow as the flow progresses downstream. The left hand columns give the downstream location (on the top line), and a vertical location relative to the critical layer (the bottom line).

Bursts		**Surrounding flow**	
location from forward restrictor	Spectral Entropy	location from forward restrictor	Spectral Entropy
2 mm downstream along critical layer	1.98	2 mm downstream, along critical layer	2.48
2 mm downstream above critical layer	2.37	2 mm downstream above critical layer	2.43
3 mm downstream along critical layer	2.02	3 mm downstream along critical layer	2.25
3 mm downstream above critical layer	2.29	3 mm downstream above critical layer	3.03
5 mm downstream along critical layer	2.37	5 mm downstream along critical layer	2.78
5 mm downstream above critical layer	2.60	5 mm downstream above critical layer	3.91

3. Spectral entropy behavior of the critical layer

By studying the spectral entropy of an entire time-series, we can determine where to locate regions of self-organization. This enables us to pinpoint within the flow the origin of various coherent structures. Hence, if one wishes to employ the characteristics of a vortex burst in a particular application, say to promote mixing or droplet breakup, then they can determine the optimum location in the flow field.

The spectral entropy was computed from the entire measured velocity time-series data at six downstream locations in the flow field. At each downstream station, measurements were taken along the critical layer and at a location slightly above the layer. This procedure is similar to the one used in Section 2, except the bursts are not excised from the time data. Also, three additional locations are included: 7, 18, and 28 mm downstream from the restrictor.

At the first three downstream stations (2, 3, and 5 mm), the power spectra of twelve measurement runs were summed together. From these power spectra we computed the spectral entropy. In Figures 3–5, we see typical time-series data for the first three stations, both along and above the critical layer. In these figures, one notes the distinct characteristics of the bursts at the different downstream locations.

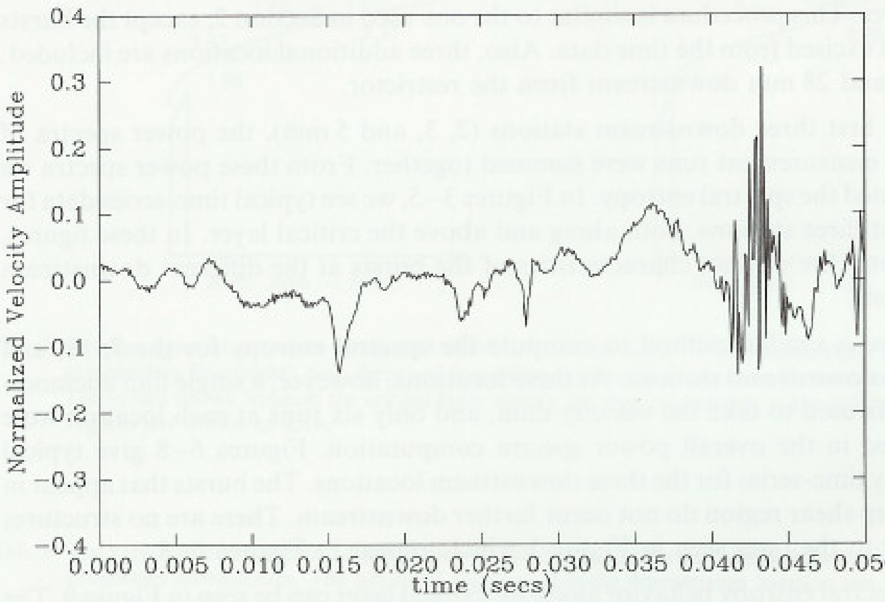
We used a similar method to compute the spectral entropy for the 7, 18, and 28 mm downstream stations. At these locations, however, a single film anemometer was used to take the velocity data, and only six runs at each location were included in the overall power spectra computation. Figures 6–8 give typical velocity time-series for the three downstream locations. The bursts that appear in the steep shear region do not occur farther downstream. There are no structures similar to the ones seen in Figure 1 which appear in Figures 6–8.

The spectral entropy behavior along the critical layer can be seen in Figure 9. The curved solid line approximately represents the critical layer as it exists in the flow cavity. At two millimeters downstream, the spectral entropy along the critical layer is at a minimum, and monotonically increases to the seven millimeter station. Due to the decrease in the spectral entropy at seven millimeters, we believe that additional vortex structures begin to form. This may explain the decrease in spectral entropy.

In the near region of the restrictor, the spectral entropy along the critical layer is significantly lower than in the region slightly above. This observation reinforces the view that the experimentally observed vortex structures are created along the critical layer. These structures then dissipate causing an increase in the spectral entropy. Farther downstream, other flow instabilities may become significant and form additional vortex structures.

Up to now, we have concentrated on the flow in the near region, downstream of the forward restrictor. If we now consider the difference between the upstream state of the flow and the location two millimeters downstream from the restrictor and along the critical layer, we note a marked change in the spectral entropy.

In experimental work performed in the upstream portion of the cavity, Hagen [11] has shown that no dominant frequency exists in this flow field. By assuming that the velocity field decomposes into mean and randomly fluctuating components, the spectral entropy reaches a maximum value, $S_{\max} = \log(N/2)$, before the flow ever reaches the cavity. Here, N is the number of data points in the measurement. For our data, $N = 512$, giving $S_{\max} \approx 5.55$. Notice then the sharp decrease in the spectral entropy from the upstream to the downstream side of the restrictor.



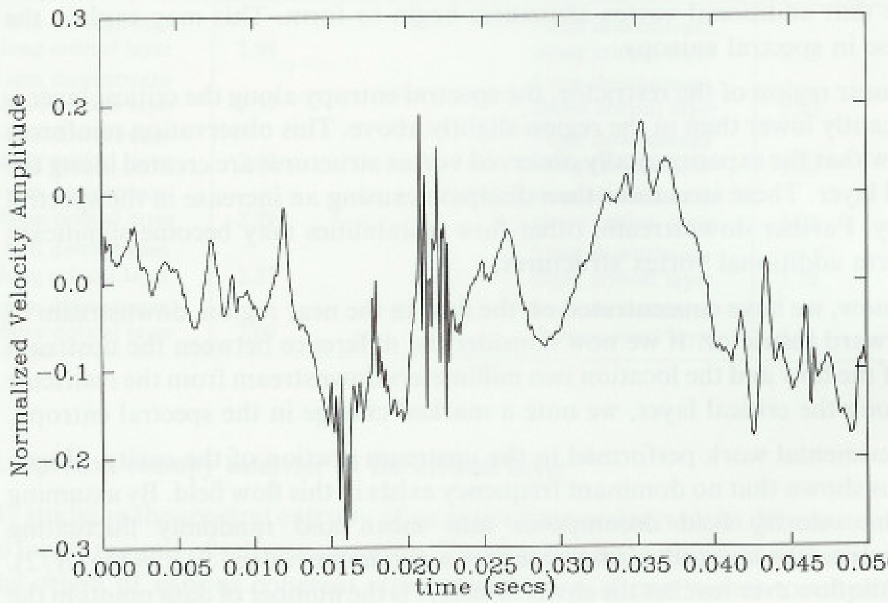
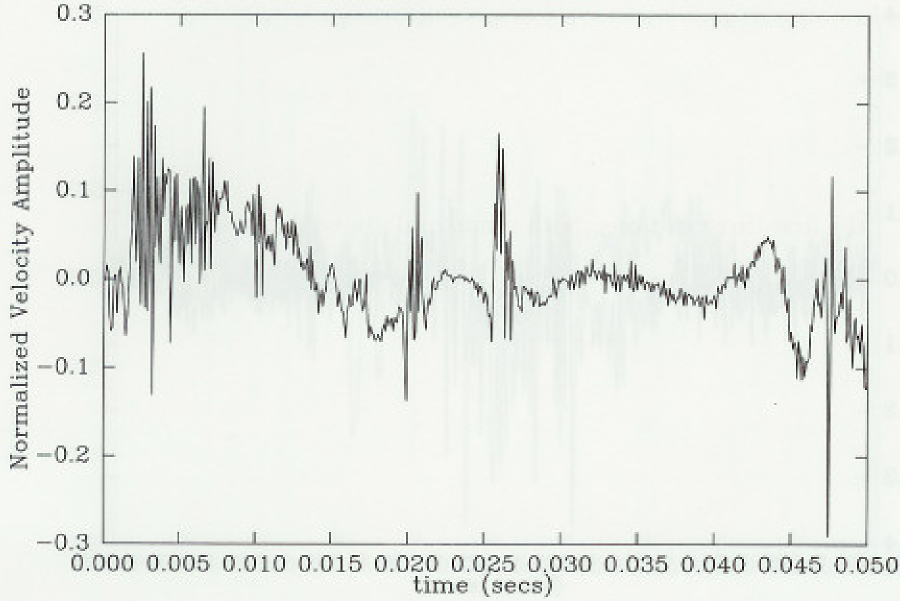


Fig. 3: Typical velocity time series, 2 millimeters from the forward restrictor. Top: Slightly (∼1 mm) above the critical layer. Below: Along the critical layer.



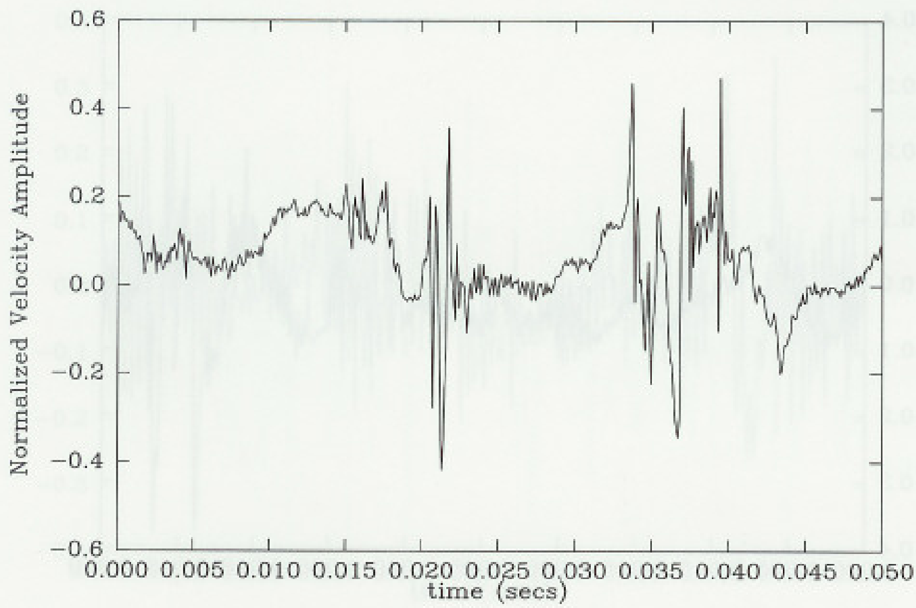


Fig. 4: Typical velocity time series, 3 millimeters from the forward restrictor. Top: Slightly (~ 1 mm) above the critical layer. Below: Along the critical layer.

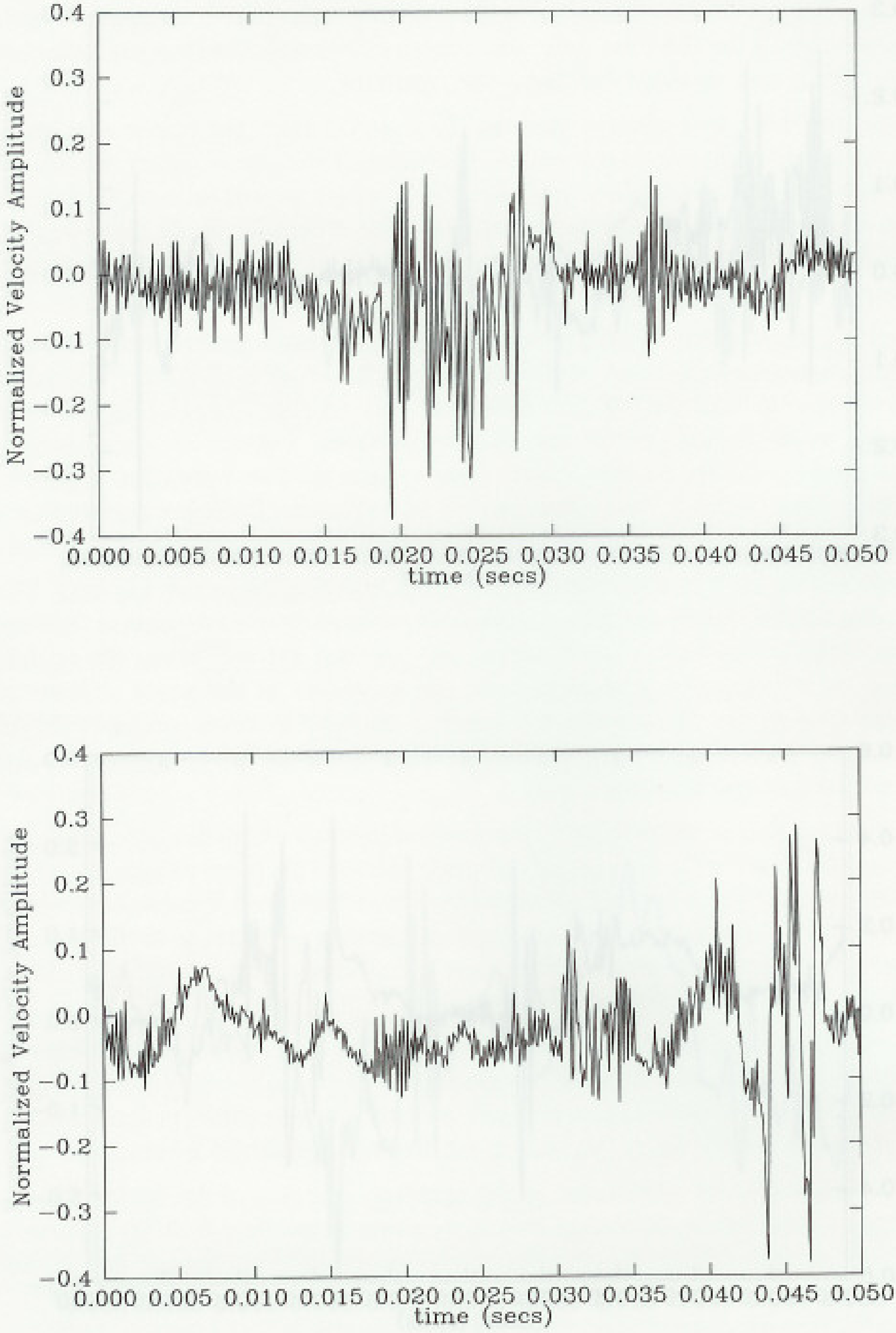
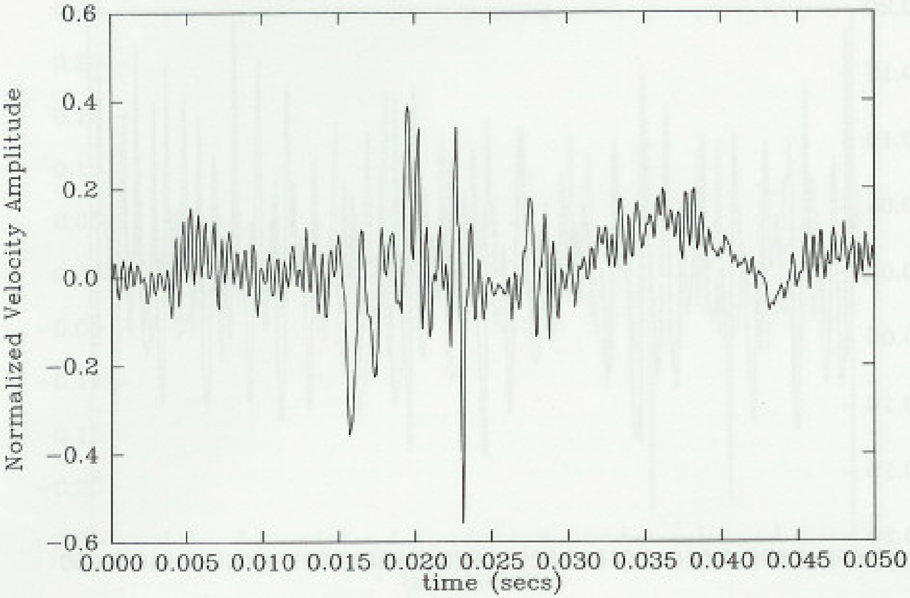


Fig. 5: Typical velocity time series, 5 millimeters from the forward restrictor. Top: Slightly (~1 mm) above the critical layer. Below: Along the critical layer.



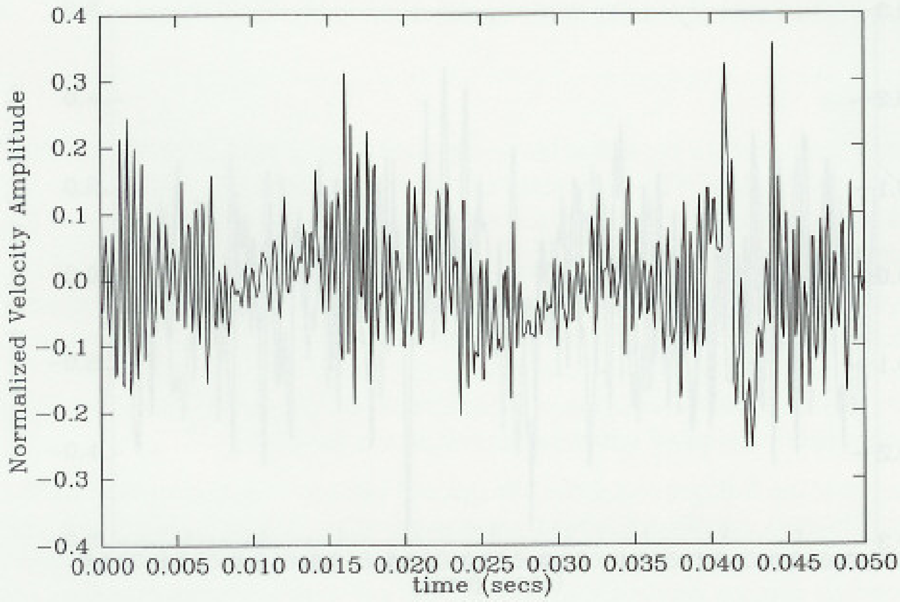
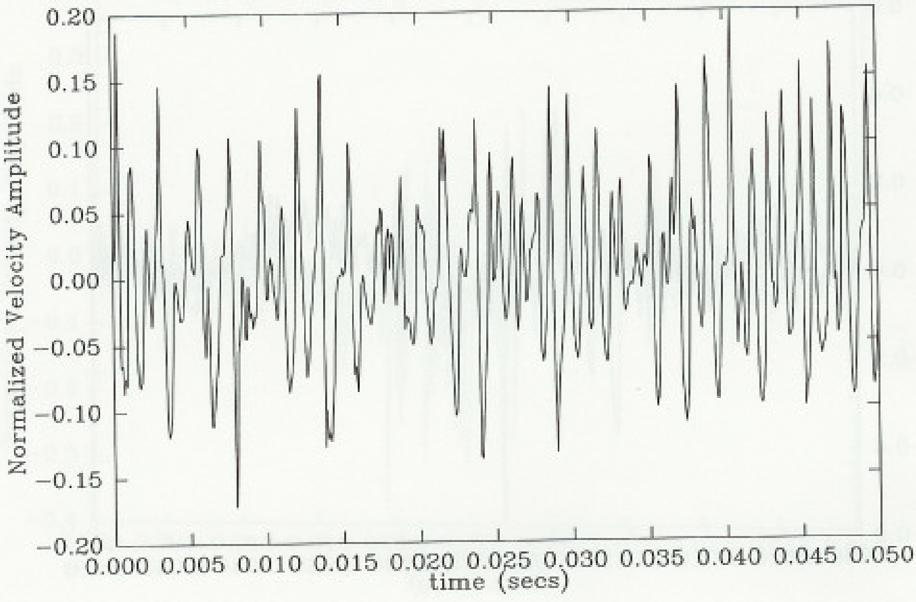


Fig. 6: Typical velocity time series, 7 millimeters from the forward restrictor. Top: Slightly (~ 1 mm) above the critical layer. Below: Along the critical layer.



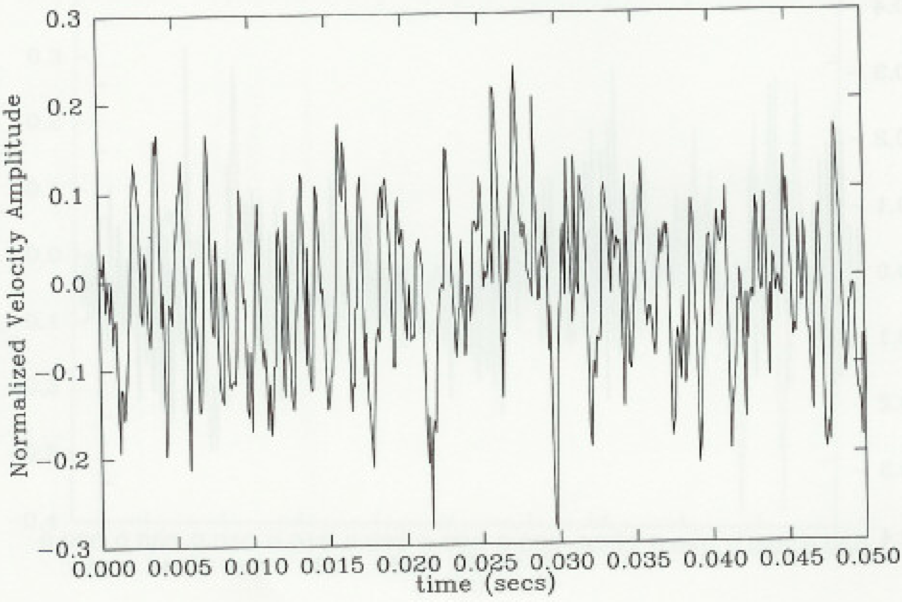


Fig. 7: Typical velocity time series, 18 millimeters from the forward restrictor. Top: Slightly (~1 mm) above the critical layer. Below: Along the critical layer.

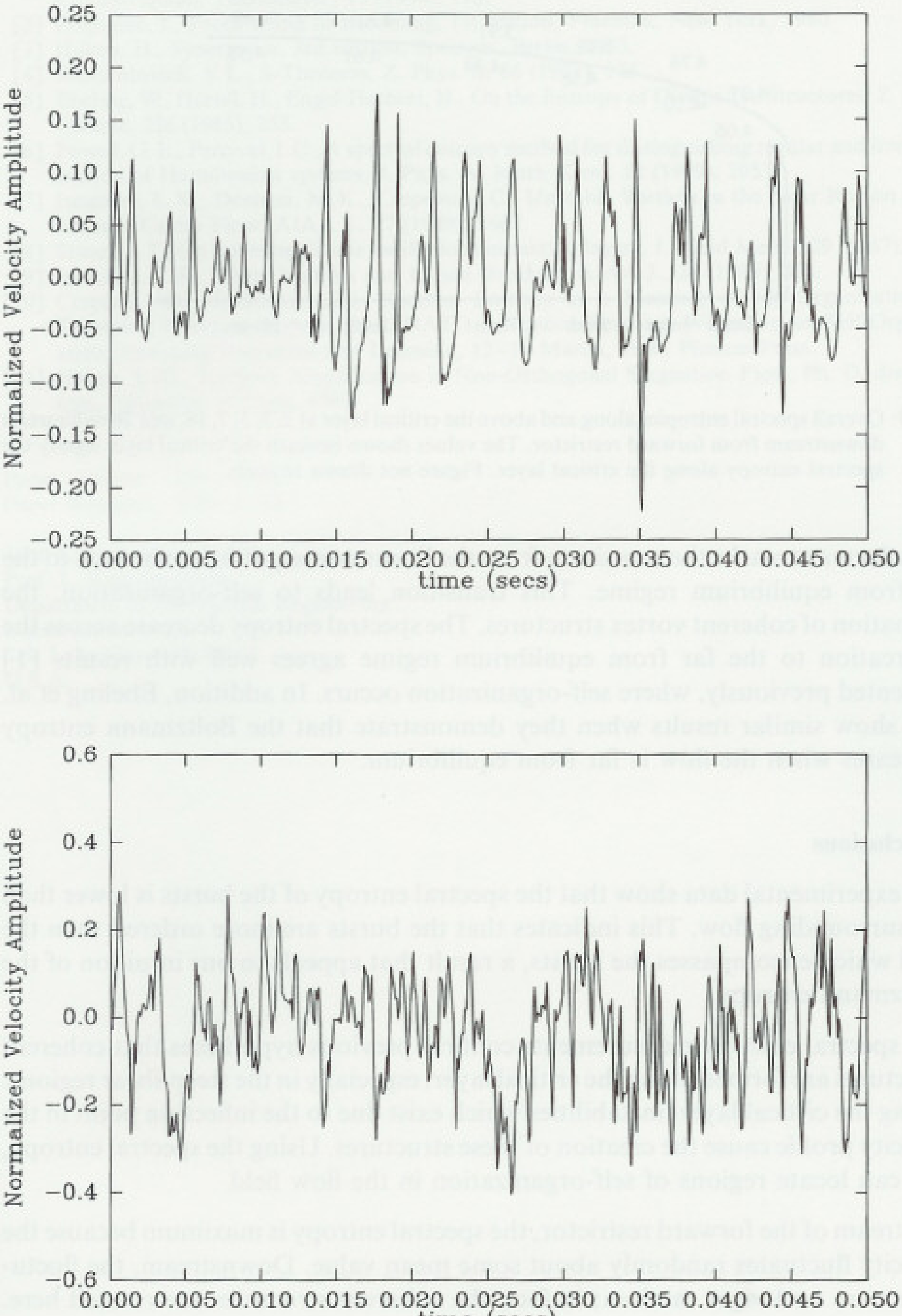


Fig. 8: Typical velocity time series, 28 millimeters from the forward restrictor. Top: Slightly (~1 mm) above the critical layer. Below: Along the critical layer.

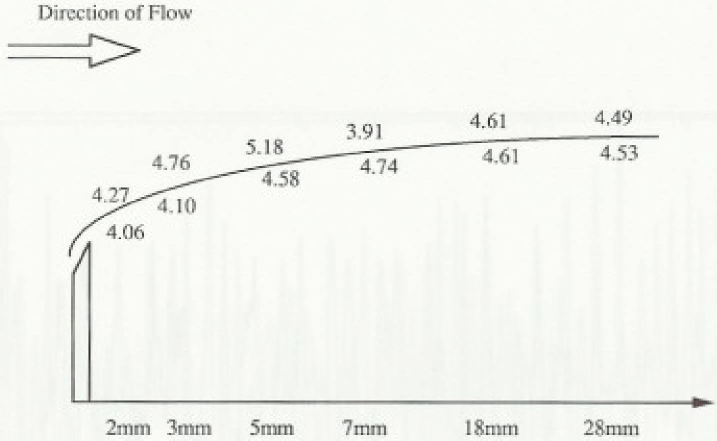


Fig. 9: Overall spectral entropies along and above the critical layer at 2, 3, 5, 7, 18, and 28 millimeters downstream from forward restrictor. The values shown beneath the critical layer signify the spectral entropy along the critical layer. Figure not drawn to scale.

This decrease marks the transition from the linear non-equilibrium branch to the far from equilibrium regime. This transition leads to self-organization, the formation of coherent vortex structures. The spectral entropy decrease across the bifurcation to the far from equilibrium regime agrees well with results [1] presented previously, where self-organization occurs. In addition, Ebeling et al. [5], show similar results when they demonstrate that the Boltzmann entropy decreases when the flow is far from equilibrium.

Conclusions

The experimental data show that the spectral entropy of the bursts is lower than the surrounding flow. This indicates that the bursts are more ordered than the fluid which encompasses the bursts, a result that appeals to our intuition of the Boltzmann entropy.

The spectral entropy measurements confirm previous hypotheses that coherent structures are formed along the critical layer, especially in the steep shear regions. Along the critical layer, instabilities which exist due to the inflection point in the velocity profile cause the creation of these structures. Using the spectral entropy, one can locate regions of self-organization in the flow field.

Upstream of the forward restrictor, the spectral entropy is maximum because the velocity fluctuates randomly about some mean value. Downstream, the fluctuations are no longer random; in fact, the coherent structures are created here. The spectral entropy decreases, indicating that self-organizational processes are occurring across the restrictor. This behavior agrees with the findings reported by Ebeling and co-workers. Farther downstream the spectral entropy increases, indicating the dissipation of the bursts into structures with lower valued energy.

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