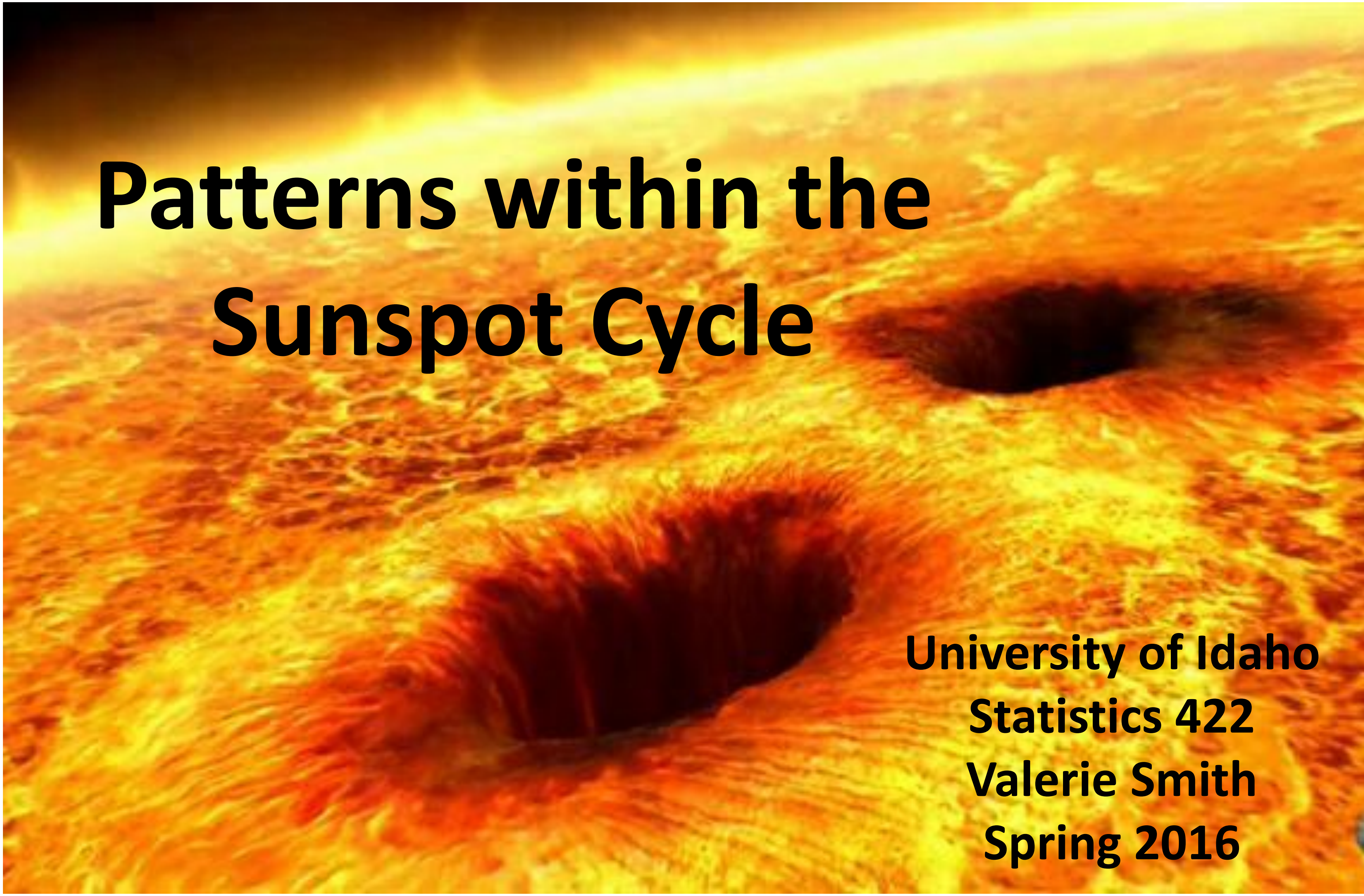
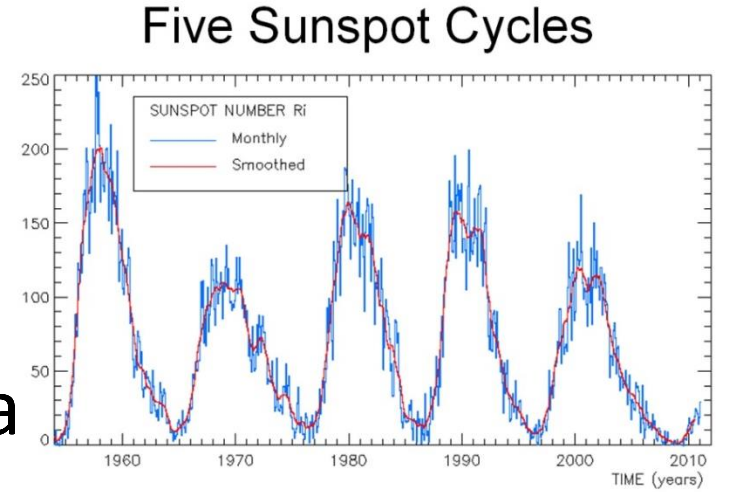
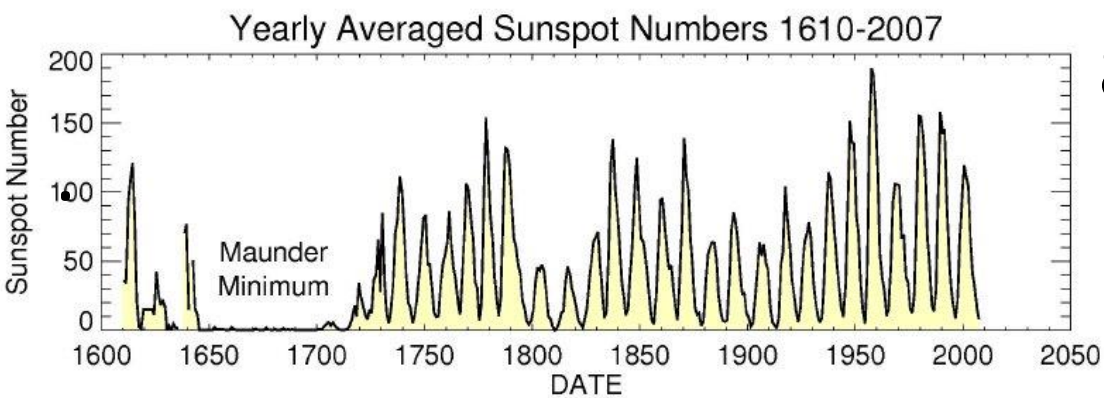

Patterns within the
Sunspot Cycle
University of Idaho
Statistics 422
Valerie Smith
Spring 2016

Sunspot Cycle



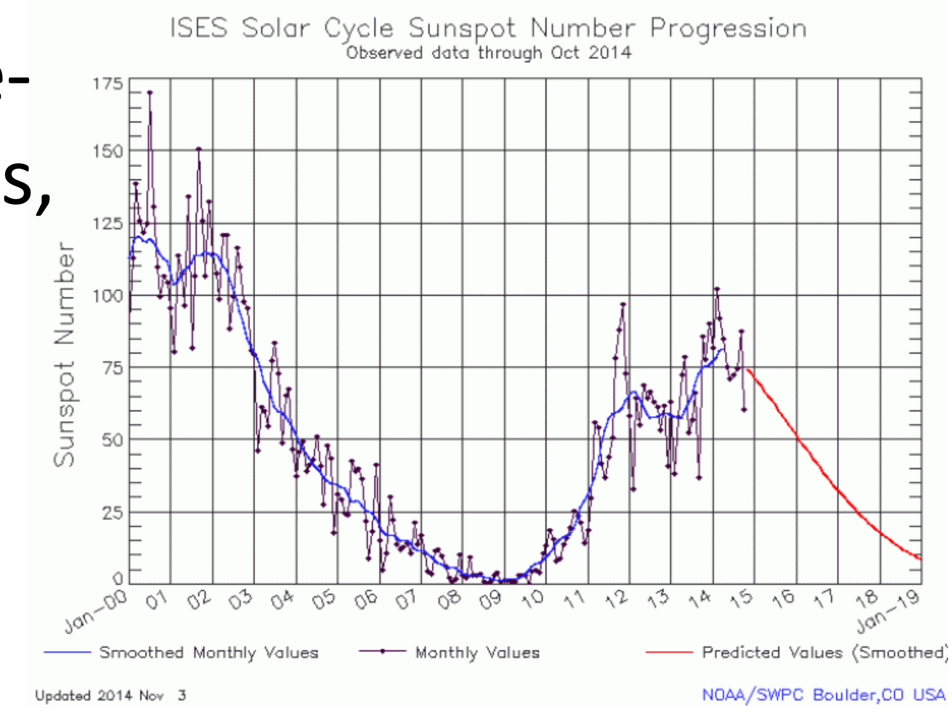
Sunspots are dark areas on the surface of the sun that form as a result of convection and magnetic activity within the sun. The number of sunspots follows a cycle of approximately 11 years①, but there are longer overall cycles as well. Dendroclimatology has shown patterns of 105, 131, 210, 232, 385, 504, 805 and 2241 years ②



Patterns within Cycles

Interestingly, there are not only larger overall patterns, but smaller-scale variations as well. Usually these variations are referred to as "noise," but what patterns exist within the noise?

In exploring the large- and small-scale patterns, multiples of seven seemed to emerge.



Large-Scale and Small-Scale Patterns

Multiples of seven in the dendroclimatology cycles are:

$105 = 7 \cdot 15,\ \ 131 \approx 7 \cdot 19,\ \ 210 = 7 \cdot 30,\ \ \ 232 \approx 7 \cdot 33,$

$385 = 7 \cdot 55,\ \ 504 = 7 \cdot 72,\ \ 805 = 7 \cdot 115,\ \ 2241 \approx 7 \cdot 320$

A smaller-scale pattern also appears in the synodic rotational period of the sun③. This period varies with latitude, but using the latitudes in which sunspots normally appear (7^o to 45^o), an average of 27.271 days* was found that fits well within the 11-year cycle: $11 \approx 27.271 \cdot 7$

Are these patterns coincidental, or are they an integral part of the small-scale variations? Is there a cycle of sevens within the daily sunspot observations?

(*27.271 was calculated using the angular velocity of the sun at an average latitude of 26^o.)

Pattern of Sevens?

To determine if there is evidence of a cycle of sevens in the daily sunspot values, the variance of a repeated systematic sample with k=7 was compared to the variance of a simple random sample of the same data. The premise was that a cycle of sevens would likely cause the variance of the repeated systematic sample to be more efficient than the variance of the simple random sample. A different sampling for the synodic rotation period (with k=27.271) was also considered, but as the k value is not an integer it would be difficult to perform the calculations in a timely manner.

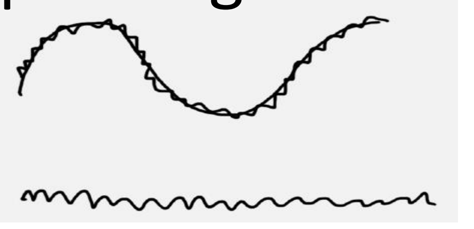
The frame used was the average number of sunspots per day from 1818 to 2016, the elements were the average number of sunspots, the sampling units were the days, and the population was the sunspot observations for each day from 1818 to 2016.

Data Restrictions

Years prior to 1849 have problems with missing data, as observations were not made consistently. To avoid the problem of missing data, the population was restricted to the years 1849 to 2016. Even with the restricted population though, when the calculations were performed in SAS a problem arose as the amount of data was too large for the program. To accommodate the limits of the program, it was necessary to further restrict the population to the years from 1950 to 2014. Data from more recent years was used, as research has shown slight differences in calculations of the number of sunspots due to technological advances[4].

Sampling the Data

Using information on the average number of sunspots per year and the dates and values of sunspot maximums and minimums, the data was divided into intervals between sunspot maximums and minimums. For each interval a quartic or cubic regression was calculated (depending on the number of years). The regressions were used to transform the data to eliminate the effect of the 11-year cycle and allow a closer look at the smaller-scale variations.



Sketch of transformation of data to eliminate effect of 11-year cycle.

With the data transformed, a simple random sample was taken of n = 250. The data was sampled again using repeated systematic random sampling with 5 samples of n = 50 and k = 7 for each.

Analysis

The simple random sampling of the number of sunspots (transformed) gave a mean of 33.33 and a variance of 9.12.

Estimate of the Population Mean
Simple Random Sampling Design
Response Variable = transspot

Estimate	Standard Error	Bound	s^2 (transspot)	Sample Size
33.3294	3.01920	6.03841	2303.42	250

The repeated systematic random sampling of the number of sunspots (transformed) gave a mean of 27.70 and a variance of 0.27.

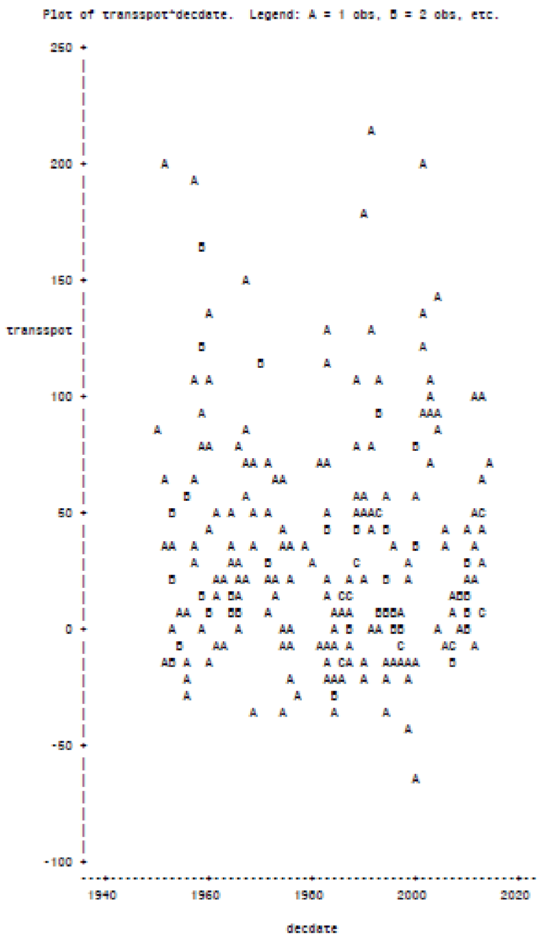
The relative efficiency of the estimators is:

$$\widehat{RE}\left(\frac{E_{sy}}{E_{SRS}}\right) = \frac{V_{SRS}}{V_{sy}} = 33.78$$

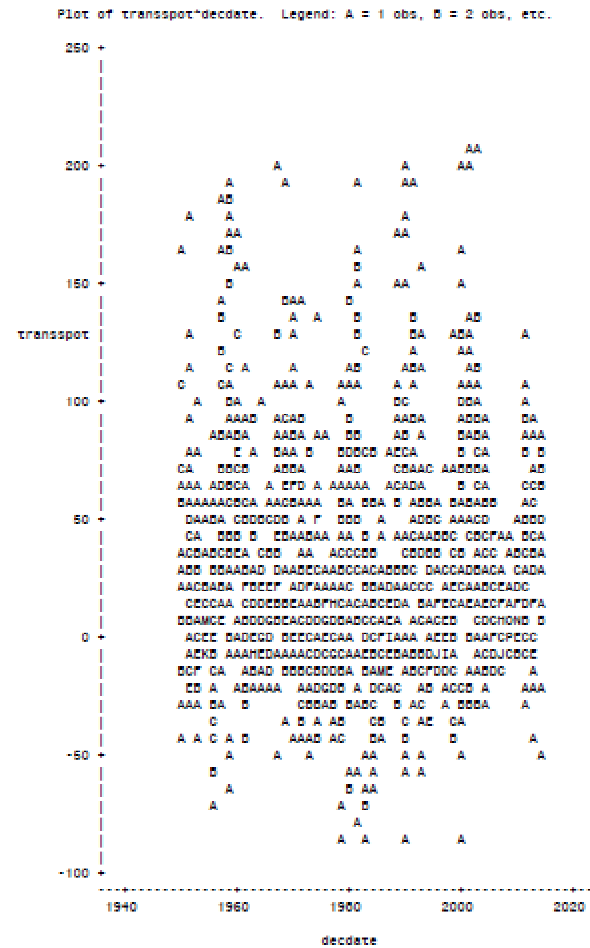
The MEANS Procedure

Analysis Variable : my			
N	Mean	Variance	Std Error
5	27.7023340	0.2665477	0.2308886

Graphs of Samples



Simple Random Sample of Transformed Average Daily Number of Sunspots



Repeated Systematic Random Sample of Transformed Average Daily Number of Sunspots

Conclusions

The high relative efficiency of the repeated systematic random sampling as compared to the simple random sampling provides evidence that above and beyond the 11-year cycle, there may be a cycle of sevens within the daily variations of the average number of sunspots. This may be related to the synodic rotational period of the sun, or there may be other factors involved such as interior dynamics of the sun or gravitational waves from planets. Further research will need to be done to reveal the complexities involved in this and other patterns.

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