

A4

Stat 301

Summer 2019

- (1) A certain market has both an express checkout line and a superexpress checkout line. Let X denote the number of customers in line at the express checkout at a particular time and let Y denote the number of customers in the line at the superexpress checkout the same time. Suppose the joint pmf of X and Y are found in the accompanying table.
- (a) What is the probability that there is exactly one customer in each line?
 - (b) What is the probability that the numbers of customers in the two lines are identical?
 - (c) Find $p_X(x)$ and $p_Y(y)$ (marginal distributions of X and Y)
 - (d) calculate:
 - (i) $E(X)$, $E(Y)$
 - (ii) $E(X^2)$, $E(Y^2)$
 - (iii) $V(X)$, $V(Y)$
 - (iv) $SD(X)$, $SD(Y)$
 - (v) $E(XY)$
 - (vi) $Cov(X, Y)$
 - (vii) $Corr(X, Y)$
 - (viii) $E(X + Y)$, $E(X - Y)$
 - (ix) $V(X + Y)$, $V(X - Y)$
 - (e) Are X and Y independent? Show work

	y	0	1	2	3
x	0	0.08	0.07	0.04	0.00
	1	0.06	0.15	0.05	0.04
	2	0.05	0.04	0.10	0.06
	3	0.00	0.03	0.04	0.07
	4	0.00	0.01	0.05	0.06

- (2) *Trial of the Pyx.* Medieval English kings devised a procedure to ensure that the coins of the realm contained the proper amount of gold. A sample of 100 of the gold coins that were cast each year was placed in a ceremonial box called the Pyx. At the chosen time, the Company of Goldsmiths jury weighed the coins. The mean weight of a coin was to be 128 grams. If the mean weight of the coins was much less than 128 grams, the jury concluded that the Master of the Mint was cheating the crown by pocketing the excess gold. If the mean weight of the coins was within 0.32 grams of the expected 128 grams (indicating that the standard deviation was 16 grams), the jury accepted the year's gold as pure. Thus the mean weight had to be between 127.68 and 128.32 grams. Anything less than 127.68 grams would be suspect and generally prescribed horrific punishments.
- (a) Define the CLT (referring to the sample mean).
 - (b) In a sample of 100 gold coins, describe the sampling distribution of the sample mean of the weight of gold coins made by the Master of the Mint. State the mean and standard deviation of the sampling distribution of the sample mean (the mean and standard error)
 - (c) What if the Master of the Mint set the mean amount of gold per coin in the population of all coins to be 127.9 ($\mu = 127.9$) grams instead of the required 128, shortchanging the crown by a tenth of a gram of gold per coin? Would the jury have noticed this? [Hint: this is the probability of the acceptance range if $\mu = 127.9$ rather than the 128 gram baseline]? Make sure to answer the question, not just compute the probability.

- (3) *Finding Dory*. Coral Reef communities are home to one-quarter of all marine plants and animals worldwide. These reefs support large fisheries by providing breeding grounds and safe havens for young fish of many species. Coral reefs are seawalls (protecting shorelines from tides, storm-surges, and hurricanes as well as produce the limestone and sand of which beaches are made), Marine scientists say that a tenth of the world's reef systems have been destroyed in recent times. At current rates of loss, almost three-quarters of the reefs could be gone in 30 years.

A particular lab studies corals and the diseases that affect them. Dr. Drew Harvell and his lab sampled sea fans at 19 randomly selected reefs along the Yucatan peninsula and diagnosed whether the animals (the sea fans) were affected by *aspergillosis*¹. In specimens collected at a depth of 40 feet at the Las Redes Reef in Akumal, Mexico, scientists found that 52% of the 104 sampled sea fans were infected with *aspergillosis*.

- (a) What are the mean (proportion, p) and standard deviation of the sampling distribution of the sample proportion (mean (p) and $se_{\hat{p}}$) of infected sea fans? What should the distribution look like (think of the definition of CLT)?
 - (b) What is probability that the sample proportion of infected sea fans is less than 50% (that is find $P(\hat{p} < .5)$)?
 - (c) What is the proportion of infected sea fans that represents the 90th percentile?
- (4) *Acme Clips*. A company packages paper clips in boxes labeled “100 pieces.” Suppose that the number of clips in boxes produced by this company has a mean of 100 and standard deviation 8. A carton consists of 64 boxes of clips. Let $\hat{\tau}$ be the total number of clips per carton (containing 64 boxes).
- (a) What are the total and standard deviation of the sampling distribution of the sample total (calculate total and se)? Describe the distribution (what should it look like?) and use shorthand notation.
 - (b) What is the probability that the *carton* (64 boxes) will contain between a total of 6250 and 6400 clips?
 - (c) What is the probability that the *carton* (64 boxes) will contain fewer than 6300 clips?
 - (d) How many clips is the bottom 5% of cartons?

¹K.M.Mullen, C.D.Harvell, A.P.Alker, D. Dube, E. Jordán-Dahlgren, J. R. Ward, and L.E. Petes, “Host range and resistance to aspergillosis in three sea fan species from the Yucatan”, *Marine Biology* (2006), Springer-Verlag.