Hypothesis Testing Rejection Criteria

The driving force for rejecting the null hypothesis \((H_0)\) is the sign of the alternative hypothesis \((H_a)\). The null represents no changes, no interactions, no differences, no effect, etc. while the alternative represents some form of a change, whether it be less than \((<)\), greater than \(>)\) or even an unspecified direction of difference \((\neq)\). Both of the approaches here are still being driven by the sign of the alternative hypothesis.

1. Critical Value approach; the null hypothesis can be rejected under the following conditions:
   
a. When \(H_a: >\)

   The null hypothesis can be rejected if and only if (iff):
   
   \[ z_{calc} \geq z_\alpha \]  where \(z_\alpha\) is the z-score associated with RIGHT tail area \(\alpha\); \(z_\alpha\) will be a positive number.

   ![Standard Normal Distribution](image)

   b. When \(H_a: <\)

   The null hypothesis can be rejected if and only if (iff):
   
   \[ z_{calc} \leq z_\alpha \]  where \(z_\alpha\) is the z-score associated with LEFT tail area \(\alpha\); \(z_\alpha\) will be a negative number.

   ![Standard Normal Distribution](image)

   When using the t distribution, you have to assume that the t-value will be negative.
c. When \( H_a: \neq \)

The null hypothesis can be rejected if and only if (iff):

\[
|z_{\text{calc}}| \geq |z_{\alpha/2}|
\]

where \( z_{\alpha/2} \) is the z-score associated with LEFT OR RIGHT tail area \( \alpha/2 \); \( z_{\alpha/2} \) will be either a negative or a positive number.

Note: These work when substituting t for z.

2. The pvalue approach; the null hypothesis can be rejected iff \( pvalue \leq \alpha \). This doesn’t change, regardless of the sign of the alternative hypothesis. However, the calculation of the pvalue is dependent on the sign of the alternative hypothesis.

a. When \( H_a: > \)

\[
pvalue = P(Z > z_{\text{calc}}) = 1 - P(Z < z_{\text{calc}})
\]

b. When \( H_a: < \)

\[
pvalue = P(Z < z_{\text{calc}})
\]

c. \( H_a: \neq \)

i. If \( z_{\text{calc}} > 0, pvalue = 2P(Z > z_{\text{calc}}) = 2[1 - P(Z < z_{\text{calc}})] \)

ii. If \( z_{\text{calc}} < 0, pvalue = 2P(Z < z_{\text{calc}}) \)

Once you calculate your pvalue, the null hypothesis can be rejected iff \( pvalue \leq \alpha \)

The pvalue calculations stated here won’t work in quite the same way with the t distribution table.