Explanation of the formal definitions

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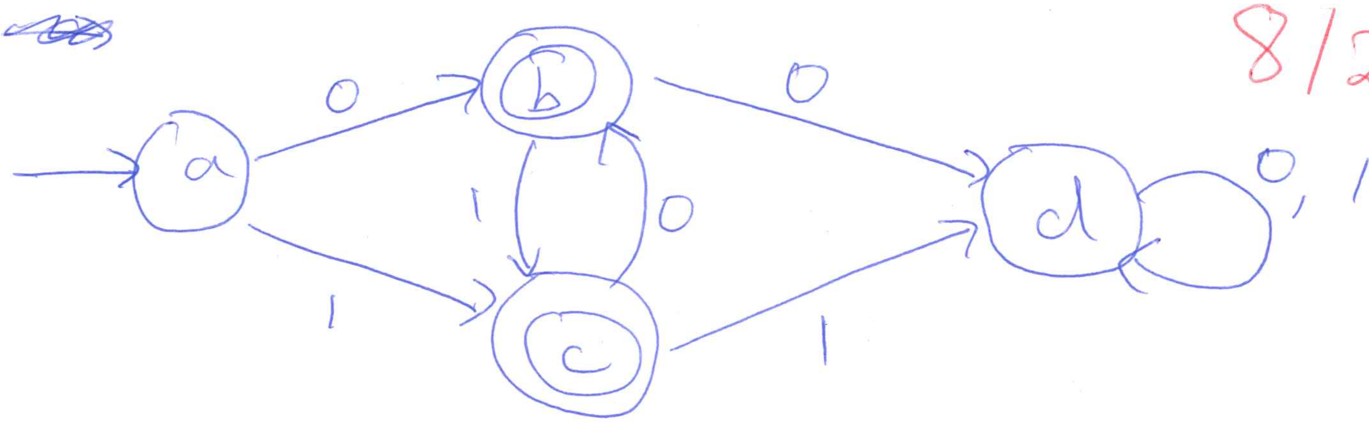

$$M = (Q, \Sigma, \delta, q_0, F)$$

$Q = \{a, b, c, d\}$

$\Sigma = \{0, 1\}$

$q_0 = a$

$F = \{b, c\}$

$\delta : Q \times \Sigma \longrightarrow Q$ is the function specified by

$\delta(a, 0) = b$

$\delta(a, 1) = c$

$\delta(b, 0) = d$

$\delta(b, 1) = c$

$\delta(c, 0) = b$

$\delta(c, 1) = d$

$\delta(d, 0) = d$

$\delta(d, 1) = d$.

Does M accept 1011?

- M accepts 1011 if M accepts 1011 starting at a.
- M accepts 1011 starting at a if M accepts 011 starting at $\delta(a, 1) = c$
- M accepts 011 starting at c if M accepts 11 starting at $\delta(c, 0) = b$
- M accepts 11 starting at b if M accepts 1 starting at $\delta(b, 1) = c$
- M accepts 1 starting at c if M accepts λ starting at d.
- M accepts λ starting at d if $d \in F$, which is not true.

So the answer to all these questions is 'No'

Next: write out a formal proof that M does what we think it does.

Prop: M accepts w if and only if ~~either~~

a) ~~w starts w~~ every 0 in w is followed by a 1 (or is the end)

AND

b) every 1 in w is followed by a 0 (or is the end).

AND

c) $w \neq \lambda$

Pf:

Lemma A: M does not accept w starting at d for any w.

Pf: ~~Base ca~~ By induction on the length (i.e. number of letters) of w.

Base case: $w = \lambda$.

By definition, since $d \notin F$, M does not accept λ starting at d.

Inductive case: $w = \ell w'$

By our inductive ~~inductive~~ hypothesis, M does not accept w' starting at d. By def'n, M accepts w starting at d if and only if M accepts w' starting at $\delta(d,\ell)$, ~~and~~ ~~for all~~ and $\delta(d,\ell)=d$ for all ℓ. So M does not accept w starting at d.

<u>Lemma B:</u> ① M accepts w starting at b ~~or c~~ if and only if $w=\lambda$ or w starts with a 1 and follows the rules in the proposition.

② M accepts w starting at c if and only if $w=\lambda$ or w starts with a 0 and follows the rules in the proposition.

<u>Pf:</u> ~~Suppose w starts w/ a 1 and follows the ~~rules~~, alternating rule.~~

Suppose $w=\lambda$. Since $b, c \in F$, M accepts w starting at b or at c.

Suppose w starts w/ a 1 and follows the alternating rule. Then M accepts $w = 1w'$ starting at b if and only if M accepts w' starting at $\delta(b,1)=c$. Since w follows the alternating rule, w' starts with a 0 and follows the alternating rule, or $w'=\lambda$. Hence, by the inductive hypothesis, M accepts w' starting at c, so M accepts w starting at b.

Similarly, if w starts w/ a 0 and follows the alternating rule, ~~M~~ M accepts w starting at c.

~~Suppos~~

Now suppose ~~w~~ w does not ~~accept~~ start with a 0 or ~~w~~ w does not follow the alternating rule. If $w = 0w'$, then $\delta(b, 0) = d$, so, by def'n, M accepts w starting at b if and only if M accepts w' starting at d. By the previous lemma, M does not accept w' starting at d, so M does not accept w starting at b.

If w ~~does not~~ starts with a 0 and does not follow the alternating rule, then w' ~~either~~ does not start w/ a 1 or does not follow the alternating rule. So, by the inductive hypothesis, M does not accept w' starting at c, so, by def'n, M does not accept w starting at b, since $\delta(b, 0) = c$.

Similarly, if w does not start with a 1 or w does not follow the alternating rule, then ~~St~~ M does not accept w starting at c. This proves the lemma. $\square$

Lemma: M accepts w starting at a if and only if $w \neq \lambda$ and w follows the alternating rule.

Pf: Homework for Monday.

Hint: Break up into cases where $w = \lambda$, w starts w/a 0, w starts w/a 1.