Equivalence of NFAs and DFAs

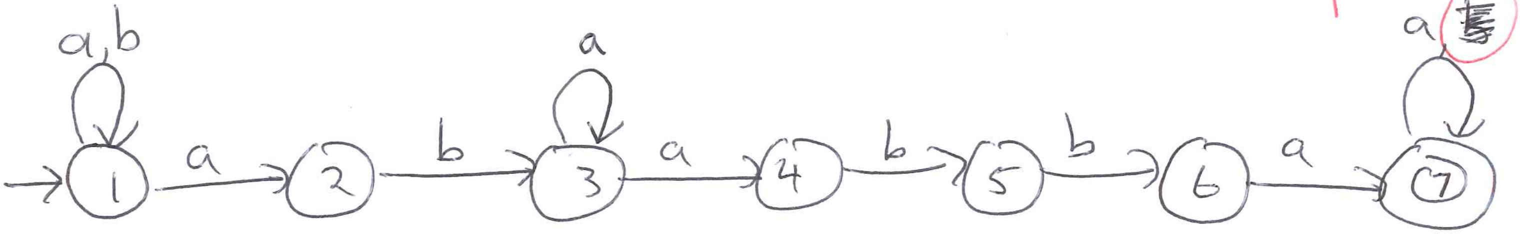
8/29

<u>Def</u>: Let M be an <u>automaton</u>. The <u>language accepted by M</u> is

$$L(M) = \{ w \in \Sigma^* \mid M \text{ accepts } w \}$$

<u>Thm</u>: For every NFA M, there exists a DFA $\overline{M}$ such that $L(\overline{M}) = L(M)$.

changed problem

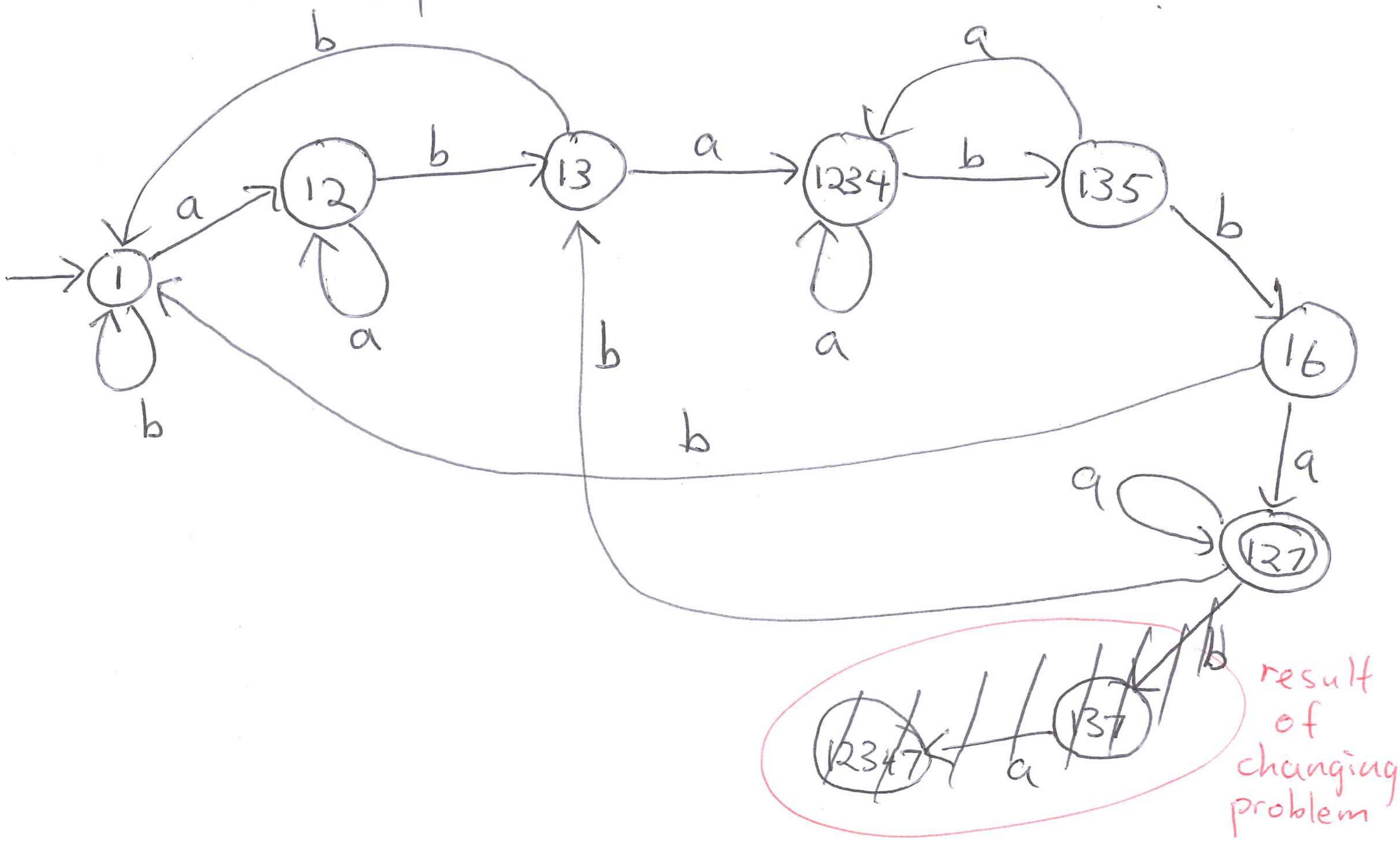


$w = ababaaba$.

How can we mechanically figure ~~this~~ out acceptance? Keep track of <u>all the states</u> we could be at after some number of letters.

So our DFA should have states that are <u>sets</u> of states of our NFA.

DFA equivalent to above NFA:



Pf: Given an NFA $M = (Q, \Sigma, \delta, q_0, F)$, construct a DFA $\overline{M} = (\overline{Q}, \Sigma, \overline{\delta}, \overline{q_0}, \overline{F})$ as follows:

$\overline{Q} = \{$ all subsets of $Q \}$

$\overline{\delta}$ is function $(\overline{Q} \times \Sigma) \longrightarrow \overline{Q}$ defined by

set of states of NFA = state of DFA

a valid transition in NFA

$$\overline{\delta}(S, \ell) = \bigcup_{q \in S} \cancel{``\delta(q, \ell)"} \{ q' \in Q \mid (q, \ell, q') \in \delta \}$$

union over all q that are elements of S

Oops- I haven't accounted for λ-transitions; will fix later. - Fixed in blue.

$$\overline{q_0} = \lambda(\{q_0\})$$

S ∩ F is not the emptyset

$$\overline{F} = \{ S \in \overline{Q} \mid S \cap F \neq \emptyset \}$$

S is a state of DFA
$\parallel$
set of states of NFA

Define for a set $S \subseteq Q$, $\lambda(S)$ is the set of states reachable from some state in S by a ~~λ-t~~ sequence of λ-transitions

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Daily HW due next Wed: $\boxed{2.3.2}$

There will be a weekly HW due next Wed - please turn in separately.