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Grade cutoffs -

50-66	C
67-83	B
84+	A

Suppose our DFA has k states.
Look at the string

$$w = a^k bba^k.$$

Note $w \in L$.
Since our DFA has k states, in reading the first k+1 letters, we have to repeat some state and go through a loop.
Consider the various possibilities for the loop:

$$w = \underbrace{a \cdots ab}_{k}\,|\,b\underbrace{a \_\_\_\_ a}_{k}$$

1 2

Possibility 1 - the loop includes m a's and not the ~~final b~~ first middle b.
If this loop is repeated, the accepted string is $a^{k+m}bba^k$. This string is not in L.

Possibility 2 - the loop includes m a's (possibly with m=0) and one b. Then the string accepted by repeating the loop is

$$\underbrace{a a \cdots a}_{k-m} \overbrace{\underbrace{a \cdots a}_{m} b}^{} \underbrace{a \cdots a}_{m} b b \underbrace{a \cdots a}_{k}$$

This string is not in L since it has an odd number of b's (and even length palindromes must have an even # of each letter

Since the loop is in the first k+1 letters, these are all the possibilities for the loop, and hence ~~must~~ our machine must accept some string not in L, a contradiction.

So L is not regular.

The Pumping Lemma (for regular languages):

Suppose L is a regular language. Then there exists a number k such that, for every string $w \in L$ with $|w| > k$ (i.e. $|w| \geq k+1$), we can decompose $w = xyz$, $|xy| \leq k+1$, $|y| > 0$ so that $xy^iz \in L$ for all $i \geq 0$.

Pf (informal): If L is regular, there is a DFA M that accepts it. ~~This~~ M has some number of states k. When M reads a string w with $|w| > k$, it must repeat a state. ~~This~~ (within the first $k+1$ letters) Call the part of w before the repeated state x, the part in the loop y, and the part after the loop z. Repeating the loop any number of times will also result in an accepted string, so

$$xy^iz \in L \text{ for all } i \geq 0.$$

General strategy for proving a language L is not regular:

chosen by your adversary

Assume L is regular. ~~Pick~~ Pick a string $w \in L$ (Your choice should depend on the number k which you don't get to choose). Show that, no matter how w is decomposed as $w = xyz$, $|xy| \leq k+1$, $|y| > 0$, you can find an i such that $xy^iz \notin L$. This contradicts the Pumping Lemma, so L can't be regular.

chosen by you

One more language:

$$L = \{a^{n^2} \mid n \in \mathbb{Z}\}$$ set of integers

(i.e. $L = \{\lambda, a, aaaa, aaaaaaaaa, \ldots\}$)

If L is regular, given k, choose $w = a^{k^2} \in L$. By the Pumping Lemma, ~~$w=xyz$~~ there exist x, y, z with $w = xyz$, $|xy| \le k+1$, ~~$|y|$~~ $y \neq \lambda$, with $xy^iz \in L$ for all i. Let $m = |y|$, and note $m \le k+1$.

Consider $xy^2z = a^{k^2+m}$

~~If~~ Is it possible for k^2+m to be a perfect square?

$k=1 \quad m \le 2 \qquad 1^2+1=2, \quad 1^2+2=3$

$k=2 \quad m \le 3 \qquad 2^2+1=5, \quad 2^2+2=6, \quad 2^2+3=7$

$k=3 \quad m \le 4 \qquad 3^2+1=10, \quad 3^2+2=11, \quad 3^2+3=12, \quad 3^2+4=13$

~~It looks like, in fact k^2+m doesn't get to the next perfect square: (i.e $k^2+m < (k+1)^2$) $\therefore$ $(k+1)^2 = k^2+2k+1$ and $m \leq k+1 < 2k+1$ (as long as $k \geq 1$)~~

Since $k \geq 1$, $k+1 < 2k+1$, so $m < 2k+1$, and $k^2+m < k^2+2k+1 = (k+1)^2$.

So $k^2 < k^2+m < (k+1)^2$, and

$k^2+m \neq n^2$ for any integer n.

So $xy^2z \notin L$, a contradiction, and L is not regular.