Useful ways to turn CFGs into equivalent CFGs

10/5/18

Goal: Monday we'll talk about "normal forms" certain restrictions on what our productions look like - we want to convert every CFG into a CFG in normal form.

To do this, we need to know ways to convert CFGs into equivalent CFGs.

Useless variables:

E.g.

$$S \to \cancel{xAbly}\ xAy \mid yBy$$

$$A \to xSx \mid Bx$$

$$B \to ySx \mid Ay \mid \lambda$$

$$C \to xBy$$

useless

How do we find these useless variables?

~~start at S~~

useful = (S)

take a useful variable, add every var it produces into the list of useful var.

2 lists of variables:
analyzed
useful

put the just analzed variable
~~in~~ into the analyzed list
repeat until every useful variable
is analyzed.

E.g. $S \to xAy \mid yBy$

$A \to xSx \mid Bx \mid \lambda$

$B \to \cancel{ySx \mid By}\ Cy$

$C \to Bx$

both useless

Here, once you get a B, you can never get rid of it (more generally, you cannot get rid of all vars)

So B is useless - you can't produce an actual string of letters from it.

How to check?

Every production ending in only letters makes the var good.

Every prod. ending in only letters and good vars makes the LHS var good.

If going through the list of prods doesn't expand list of good ~~lette~~ vars, you are done.

Step 0 for simplifying grammars:

Get rid of all useless variables.

Getting rid of λ-productions:

E.g. $S \to xAy \mid yBy \mid yy$

$A \to xSx \mid Bx \mid x$

$B \to ySx \mid Ay \mid \cancel{\lambda}$

Add ~~pro~~ all the productions that do

$\sim \to \sim B \sim \to \sim\sim$

as one step productions

~~Since~~ If I have a derivation

$S \to xAy \to xBxy \to xxy$

Change the grammar so that I have

$S \to xAy \to xxy$

General principle:

If I have a production

$A \to w$ (A a var, w a string of vars & letters)

I can get rid of the production at the price of adding an extra production to every production that produces A.

Careful

$$A \to xBxBy \mid xxBy \mid xBxy \mid xxy$$
$$B \to \cancel{\lambda} \mid \ldots$$

all the possibilities for subbing λ for B

Getting rid of $A \to B$ productions (unit productions)

to get rid of this, add

<u>E.g.</u>
$$S \to xAy \mid Bx$$
$$A \to \cancel{B} \mid xS \mid \boxed{xAx \mid By \mid Sy \mid \lambda}$$
$$B \to xAx \mid By \mid Sy \mid \lambda$$

Careful - we can have a chain of unit productions

$$S \to xAy \mid Bx$$
$$A \to B \mid xS$$
$$B \to xAx \mid C \mid \lambda$$
$$C \to By \mid A$$

A, B, + C can all go to each other, so all prods of any of them are prods of all.

$S \longrightarrow x\,\cancel{B}\,y \mid \Xi x$

$\Xi \longrightarrow xS \mid x\Xi x \mid \lambda \mid \Xi y$

~~Sm~~ $\Xi = \cancel{B}C \longleftarrow$ A on top of B on top of C.

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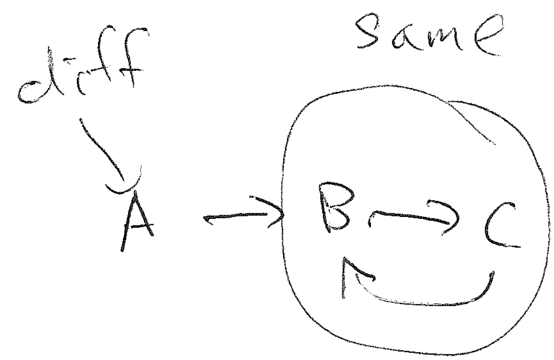
or a non loop:

$S \rightarrow xAy \mid Bx$

$A \rightarrow B \mid xS$

$B \rightarrow xAx \mid C \mid \lambda$

$C \rightarrow By \mid B$



$S \longrightarrow xAy \mid \Xi x$

$A \longrightarrow xS \mid xAx \mid \lambda \mid \Xi y$

$\Xi \longrightarrow xAx \mid \lambda \mid \Xi y$

merge all Prods of B + C, add things B/C produce as productions of A.