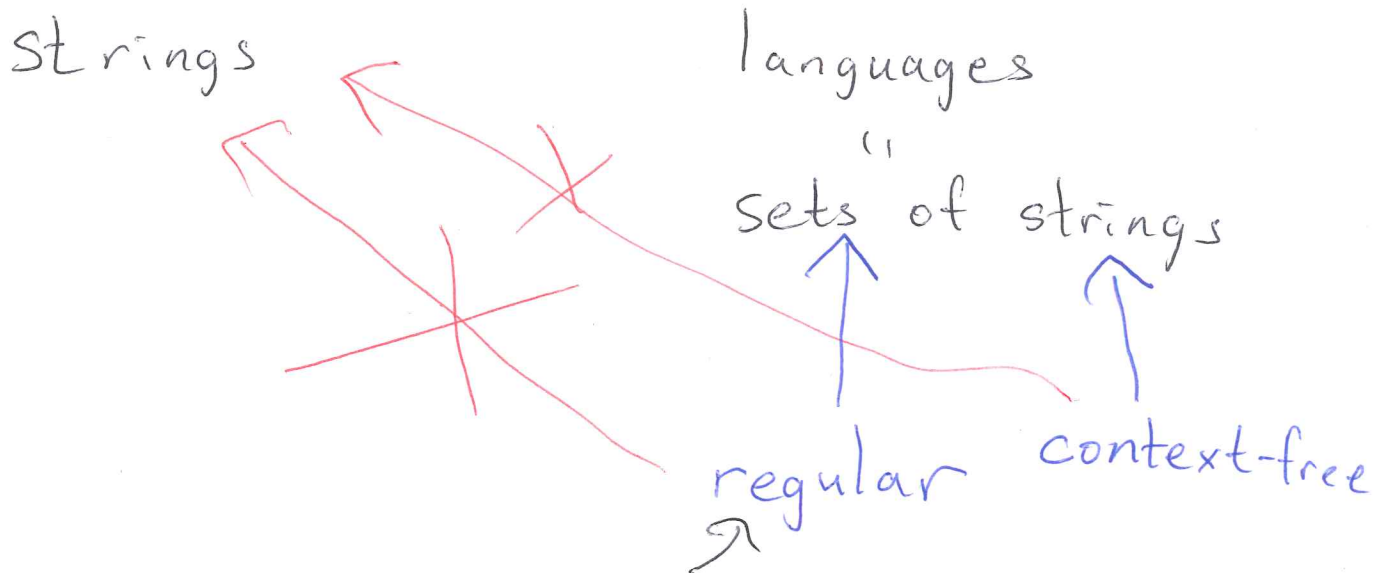


Comments on HW 3:

10/12

Careful about what "type" your mathematical objects have and what "type" your operations and predicates take.

Two types:



Not a property inherited from some property on individual strings.

Some operators are overloaded:

$w$  - string,  $|w|$  - # letters in string (by def'n is always finite)

$L$  - language,  $|L|$  - # strings in the language (which could be infinite)

concatenation:

of 2 strings  $\rightarrow$  produces a string

of 2 languages  $\rightarrow$  produces a language.

Catch your own type errors!

Pushdown automata:

Def'n of acceptance:

Recursive def'n:

Def: A PDA  $M$  accepts  $w$  from state  $q$  with  $u$  on the stack if

1)  $w \in \lambda$  and  $q \in F$ , or

2)  $w = xy$ ,  $u = st$ ,  $(x \in \Sigma \cup \{\lambda\}, y \in \Sigma^*, s \in \Gamma, t \in \Gamma^*)$ ,  
*there exist  $x, y, s, t$ ,*

*implicit convention:  
top of stack is left side of stack string.*

$[(q, x, s), (q', s')] \in \delta$

$M$  accepts  $y$  from state  $q'$  with  $s't$  on the stack.

2, rewritten) There exist  $x \in \Sigma \cup \{\epsilon\}$ ,  
 $y \in \Sigma^*$ ,  $s \in \Gamma$ ,  $t \in \Gamma^*$ ,  $q' \in Q$ ,  $s' \in \Gamma^*$ ,  
 $[(q, x, s), (q', s')] \in \delta$

such that

$w = xy$ ,  $u = st$ ,  $M$  accepts  $y$  from  
state  $q'$  with  $s't$  on the stack.

Iterative Definition:

Def'n: A PDA  $M$  accepts  $w$  if  
it accepts  $w$  from state  $q_0$  with  
 $z$  on the stack.

Iterative definition:

Def'n: Define the function  ~~$\delta$~~

$$\delta^* : (Q \times \Sigma^* \times \Gamma^*) \rightarrow \mathcal{P}(Q \times \Gamma^*)$$

power set;  
i.e. set of all subsets  
of  $Q \times \Gamma^*$

by:

$$\delta^*(q, w, u) = \bigcup_{\substack{w=xy \\ u=st}} \delta^*(q', y, s't) \quad \cancel{u(q, a)}$$

$$\cancel{[(q, x, s), (q', s')] \in \delta}$$

$$\begin{array}{c} \bigcup (q, u) \\ \uparrow \\ \text{if } w = \lambda \end{array}$$

Def'n: A PDA  $M$  accepts  $w$  if there exists a final state  $f \in F$  and some stack string  $g \in T^*$  s.t.

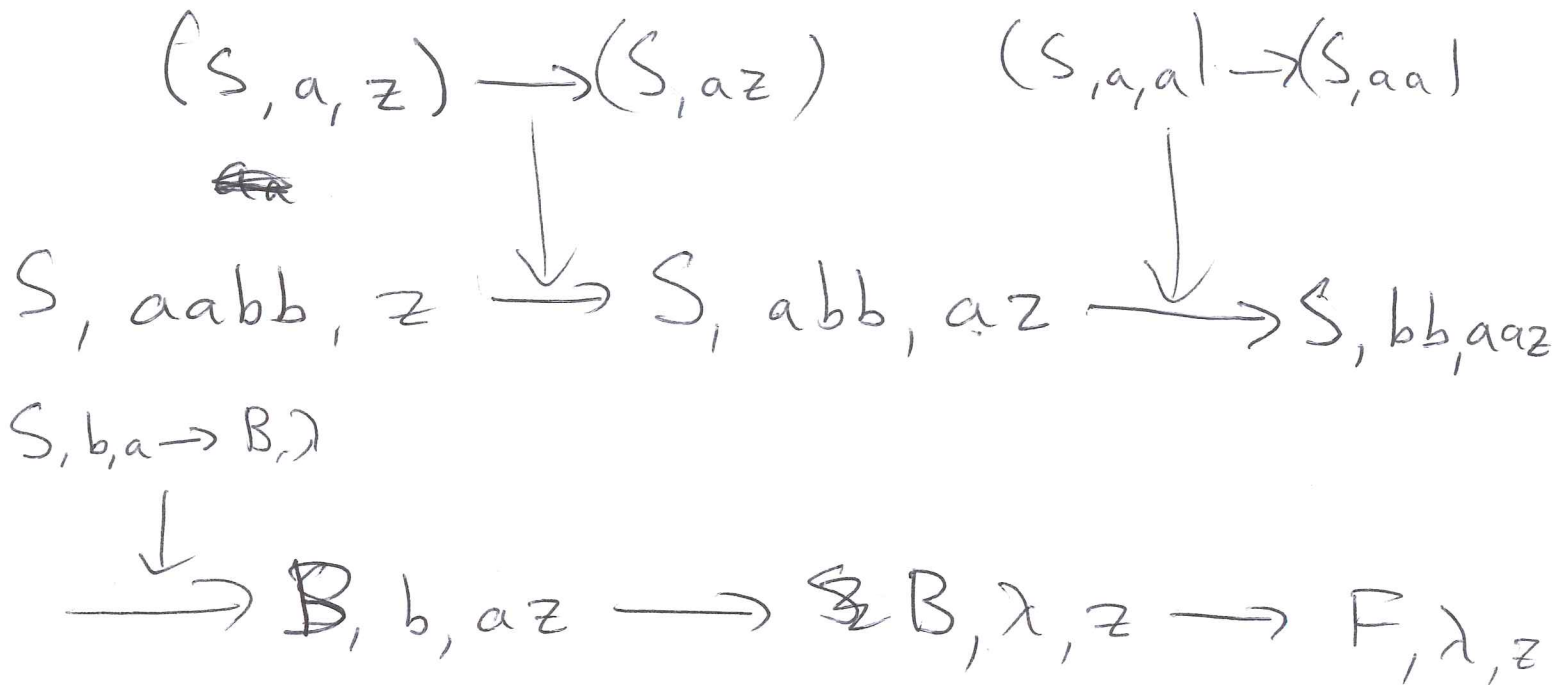
$$(f, g) \in \delta^*(q_0, w, z)$$

the set of places we can get to.

Example of how the def's work:

Take  $M$  from last time accepting  $\{a^n b^n \mid n \in \mathbb{Z}_{>0}\}$

Why does  $M$  accept  $aabb$ ?



We know  $M$  ~~acc~~

1) accepts  $\lambda$  starting from  $F$  w/  $z$  on stack,  
~~2) accept~~ (by part (i) of def'n)

2) accepts  $\lambda$  starting from  $B$  w/  $z$  on stack  
 because  $\lambda = \lambda$ ,  $z = z\lambda$ ,  
 $[(B, \lambda, z), (F, z)] \in \delta$ , and ~~part~~  
 statement (i),

3) accepts  $b^w$  starting from  $B$  with  $az$  on the stack, because ~~we~~  
~~we~~  $[(B, b, a), (B, \lambda)] \in \delta,$

$$\underbrace{b}_w = \underbrace{b}_x \underbrace{\lambda}_y, \quad \underbrace{az}_w = \underbrace{a}_s \underbrace{z}_t$$

and  $M$  accepts  $\lambda$  from state  $B$   
 with  $z$  on the stack by statement (2)

⋮

6) accepts  $aabb$  starting from  $S$  with  
 $z$  on the stack because ...