10/17

Last time - given a CFG, construct a PDA that accepts the language it generates.

set of strings

Today: Given a PDA, create a CFG that generates the language it accepts.

Key: We want our scheme to give our variables in CFG meaning in terms of our PDA.

Idea: $V = Q \times \Gamma \times Q$

variables $\longleftrightarrow$ (state, stack ~~var~~ letter, state)

The variable (q, A, q')

will be able to produce (in many steps) any string the PDA accept starting at q, ending at q', taking a single A off the stack (and ~~making no other changes to the stack~~ ending up with the same stack otherwise)

I'll insist my PDA is constructed so that it removes the whole stack (including the bottom of stack symbol) at the end, and that it has only one final state.

Start variable for the CFG (q_0, Z, q_f)

start state — bottom of stack — the only final state

Productions: ~~Transitions~~: Given a transition δ for the PDA,

$$\delta = (q, \ell, A) \longrightarrow (q', X)$$

state — letter we read — stack letter we pop — new state — what we push

1) If $X = \lambda$, then we get a prod.

$$(q, A, q') \longrightarrow \ell$$

2) If $X \neq \lambda$, then... for every state q''

$$(q, A, q'') \longrightarrow$$

Getting from q to q'' pulling A off the stack ~~is equivalent~~ can be done by eating ℓ, then getting from q' to q'' pulling $\underline{X}$ off the stack.

This gives ~~tra~~ productions

$$(q, A, q'') \longrightarrow \ell\,(q', X_1, q_1)(q_1, X_2, q_2) \cdots (q_{k-1}, X_k, q'')$$

where $\underline{X} = X_1 X_2 \cdots X_k$, and we need this production for every choice of $q_1, \ldots, q_{k-1}, q''$

Example:

Transitions in PDA:

(1) $(q_0, a, Z) \longrightarrow (q_0, AZ)$

(2) $(q_0, a, A) \longrightarrow (q_0, AA)$

(3) $(q_0, b, A) \longrightarrow (q_1, \lambda)$

(4) $(q_1, b, A) \longrightarrow (q_1, \lambda)$

(5) $(q_1, \lambda, Z) \longrightarrow (q_2, \lambda)$

Productions in CFG:

$(q_0, A, q_1) \rightarrow b$ from (3)

$(q_1, A, q_1) \rightarrow b$ from (4)

$(q_1, Z, q_2) \rightarrow \lambda$ from (5)

from(1)

$(q_0, Z, q_0) \rightarrow a(q_0, A, q_0)(q_0, Z, q_0)$

$\rightarrow a(q_0, A, q_1)(q_1, Z, q_0)$

$\rightarrow a(q_0, A, q_2)(q_2, Z, q_0)$

$(q_0, Z, q_1) \rightarrow a(q_0, A, q_0)(q_0, Z, q_1)$

$\rightarrow a(q_0, A, q_1)(q_1, Z, q_1)$

$\rightarrow a(q_0, A, q_2)(q_2, Z, q_1)$

$\cancel{(q_0, Z, q_2)}$

$(q_0, Z, q_2) \rightarrow a(q_0, A, q_0)(q_0, Z, q_2)$

$\mid a(q_0, A, q_1)(q_1, Z, q_2)$

$\mid a(q_0, A, q_2)(q_2, Z, q_2)$

$$(q_0, A, q_0) \longrightarrow a(q_0, A, q_0)(q_0, A, q_0)$$
$$\mid a(q_0, A, q_1)(q_1, A, q_0)$$
$$\mid a(q_0, A, q_2)(q_2, A, q_0)$$

+ 6 more productions