

Grade cutoffs:

10/31

~~52-54~~
67

C

~~68-70~~
83

B

84+

A

(2 pts were
for everyone
b/c only 98
pts were possible)

Turing machines

Def: A Turing machine M is a tuple

$$M = (Q, \Sigma, \Gamma, q_0, F, \square, \delta)$$

where:

Q is a finite set (of states)

~~Σ is ~~the~~ a finite set~~

Γ is a finite set (of the tape
alphabet)

$\Sigma \subseteq \Gamma$ is (the ~~at~~ input alphabet)

$q_0 \in Q$ (the initial state)

$F \subseteq Q$ (the final states)

$\square \in T$ (the blank symbol)

δ is a function

$$Q \times T \longrightarrow (Q \times T \times \{L, R\}) \cup \{\#\}$$

Def: An instantaneous description S of a Turing machine is an element \bullet

$$S \in T^* \times Q \times T^*$$

The instantaneous descriptions

(w, q, w') and $(\square w, q, w')$
are considered the same, as are

(w, q, w') and $(w, q, w' \square)$

for all $w, w' \in T^*$, $q \in Q$.

Def: ~~$(w, q, w) \vdash (v, q, v)$~~

$(u, q, v) \vdash (u', q', v')$

(u, q, v) goes to (u', q', v') in one step.

$$\text{if } v = lv'',$$

$$\delta(q, l) = (q', l', R),$$

$$u' = ul', v' = v''$$

$$l, l', l'' \in \Gamma$$

or

$$\delta(q, l) = (q', l', L),$$

$$u = u''l'', \text{ then}$$

$$u' = u'', v' = l''l'v''$$

$$u, u', v, v', v'' \in \Gamma^*$$

Def: $(u, q, v) \vdash (u', q', v')$

(u, q, v) goes to (u', q', v') in some number of steps if there exist

$$\underbrace{u_1, \dots, u_k, v_1, \dots, v_k, q_1, \dots, q_k}_{\Gamma^*} \quad \underbrace{q}_{Q}$$

s.t. $(u_i, q_i, v_i) \vdash (u_{i+1}, q_{i+1}, v_{i+1})$

for all i

and

$$u = u_1, q = q_1, v = v_1, u' = u_k, q' = q_k, v' = v_k$$

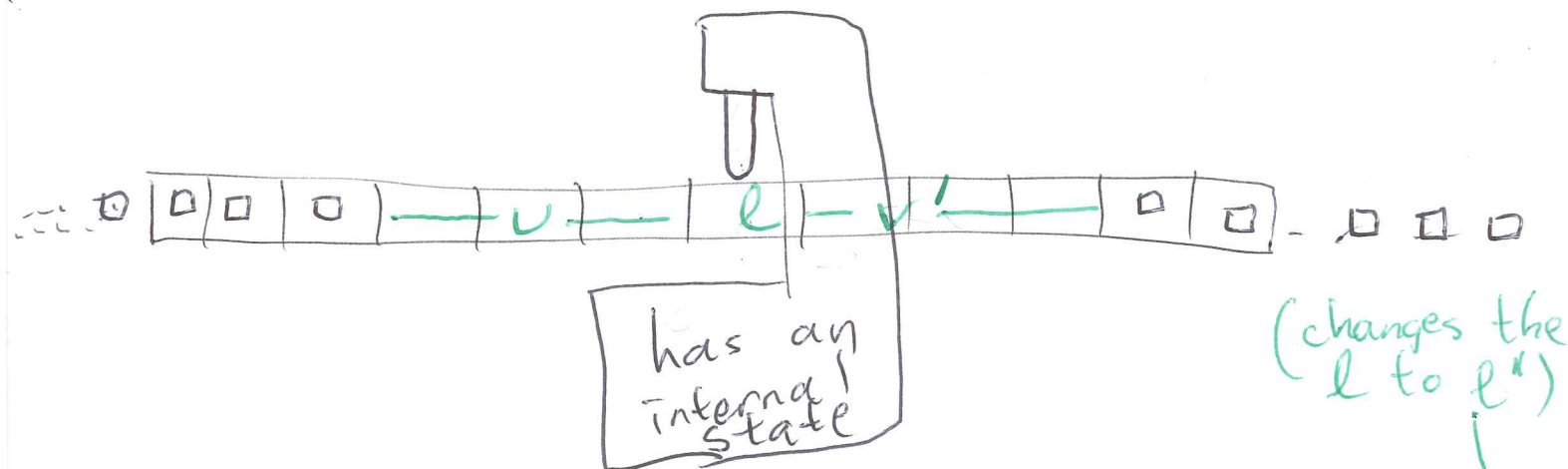
Def: A computation halts at the instantaneous description

$$(u, q, v) \text{ if } v = lv',$$

$$\delta(q, l) = \mathcal{H}.$$

Def: M accepts a string $w \in \Sigma^*$ if there exists a final state $q_f \in F$, strings $u, v \in \Sigma^*$ if $(\square, q_0, w) \models (u, q_f, v)$ and M halts at (u, q_f, v) .

Picture of Turing machine



The machine reads the letter l , and depending on its internal state q , writes l' , changes to state q' , moves itself L or R,

In the instantaneous description:

(u, q, v) ,

u is the string to the left of the head on the tape

v is the string to the right, including where the head is.

q is current state.