Review - Pumping lemma for ~~R~~ CFLs

11/28/18

From HW 9:

Show $L = \{a^k b^j c^n \mid n = jk\}$
is not context-free.

Pumping lemma: Suppose L is a context-free language. Then there exists m such that, for all $w \in L$, with $|w| \geq m$, there exist u, v, x, y, z, with $w = uvxyz$, $|vxy| \leq m$, $|vy| \neq 0$, such that $w_i = uv^ixy^iz \in L$ for all i.

To show L is not context free, we have to show some string in L cannot be pumped.

Given m, Let $w = a^m b^m c^{m^2}$. We show w cannot be pumped. Consider all the possible ways we have $w = uvxyz$ with $|vxy| \leq m$. There are several cases.

Case 1: $vxy = a^r$ for some $r \le m$.

Then $v = a^s$, $y = a^t$ $(s+t \le r,\ s+t > 0)$.

Hence $w_2 = uv^2xy^2z = a^{m+s+t} b^m c^{m^2}$.

Since $(m+s+t)m \ne m^2$, $w_2 \notin L$.

Case 2: $vxy = b^r$ for some $r \le m$,

Then $vy = b^s$ for some s, $0 < s \le r$.

Hence $w_2 = uv^2xy^2z = a^m b^{m+s} c^{m^2}$, and

since $m(m+s) \ne m^2$, $w_2 \notin L$.

Case 3: $vxy = c^r$ for some r, $0 < r \le m$.

Then $vy = c^s$ for some s, $0 < s \le r$)

and $w_2 = uv^2xy^2z = a^m b^m c^{m^2+s} \notin L$. ~~for~~

Case 4: $vxy = a^r b^s$ for some r, s, $0 < r, s \le m$,

$r + s \le m$. Then $v = a^{r'}$, $y = b^{s'}$ is the only case we have to handle, since if either v or y had both a's and b's, then $w_2 \notin L(a^* b^* c^*)$ and hence not in L, and if vy had only a's or only b's, we are effectively in Case 1 or 2. Then $w_2 = uv^2xy^2z = a^{m+r'}b^{m+s'}c^{m^2} \notin L$.

Case 5: $vxy = b^r c^s$. As in Case 4, $v = b^{r'}$, $y = c^{s'}$. Hence

$$w_2 = uv^2xy^2z = a^m b^{m+r'} c^{m^2+s'}$$

Could this be in L?

~~We have~~ $\cancel{m(m+r') \neq m^2+s'}$

$$\cancel{m^2+mr' \neq m^2+s'}$$

$$\cancel{m > s'}$$

$$\cancel{r > 1}$$

$$\cancel{mr > s'}$$

Try an example $m=15$, $r'=1$, $s'=15$ oh wait s' can't be that big

We have $m > s'$, $r' \geq 1$, so

$$mr' > s', \text{ and}$$

$$m^2+mr' > m^2+s', \text{ so}$$

$$m(m+r') \neq m^2+s'.$$

Hence $w_2 \notin L$.

Note there is no case where vxy has a's & c's since $|vxy| \leq m$.

Proving some other problems/languages are not T-D (i.e. not recursive):

Last time we proved the halting problem/lang. H is not T-D.

Suppose we have some other problem/language ~~that~~ L ~~is~~ that we want to prove is not T-D.

<u>General strategy</u>: Assume for contradiction L is T-D, i.e. there exists a TM M that decides L. Find a way to modify M (or include M in a bigger machine) to ~~solve~~ ~~(decide~~ create a machine $\hat{M}$ that decides H. We know H is not T-D, so this is a contradiction, so L is not T-D.

i.e. find a way to change ~~an instanc~~ input to H to input for L (that gives the same answer)