Homework 7 (CS/Math 385)

due October 30, 2019

Universal Rule for all homework: Unless otherwise stated, all solutions must include a brief proof that they are correct. For problems that involve following a procedure we have discussed in class, showing the steps you took and giving an indication of what was done at each step (for example writing that "State q_3 was removed to obtain the following equivalent super-NFA.") suffices to satisfy the rule.

Section 7.1: 6j, 19

Section 7.2: 9

(1) Consider the grammar with a single variable S and production $S \to ab \mid aSb \mid SS$.
  (a) Find an equivalent grammar in Chomsky Normal Form.
  (b) Use the method described in class to construct a pushdown automaton accepting the language generated by this grammar.
  (c) Find a leftmost derivation for the string $abaababb$ for this grammar.
  (d) Find the corresponding sequence of instantaneous descriptions for the PDA that shows this string is accepted.

(2) Consider the grammar $S \to AB|CD$, $A \to a|AA$, $B \to b$, $C \to AA$, $D \to AB$. (This is essentially the grammar you worked with in Homework 5, converted to Chomsky Normal Form in such a way as to maintain the ambiguity.)
  (a) Show that this grammar is ambiguous by finding two different ways to generate $aaab$.
  (b) Use the method described in class to construct a pushdown automaton accepting the language generated by this grammar.
  (c) Find the corresponding sequence of instantaneous descriptions for the PDA for each of the two different ways to generate $aaab$.

(3) Consider the pushdown automaton with the following transitions.

$$(q_0, a, z) \to (q_1, Az)$$

$$(q_1, a, A) \to (q_1, AA)$$

$$(q_1, b, A) \to (q_2, \lambda)$$

$$(q_2, b, A) \to (q_2, \lambda)$$

$$(q_2, \lambda, z) \to (q_f, \lambda)$$

(This is the automaton we looked at on Oct. 21.)
  (a) Find the sequence of instantaneous descriptions for this automaton in accepting the string ab and the corresponding derivation in the context-free grammar that we created from the pushdown automaton. (Look at the notes from Oct. 21.)
  (b) Do the same for the string $aaabbb$. Use this and the derivation for the string $aabb$ that we did in class to generalize to a derivation for the string $a^n b^n$ for any $n \geq 2$.
  (c) Since these are the only strings in the language, this tells you what the useful productions are in this grammar. Write down all of them.

(4) Suppose we have a nondeterministic finite automaton M We can create a pushdown automaton $\hat{M}$ with only one final state that is entered when the stack is empty as follows:

- Every transition of M is a transition of $\hat{M}$, where the only thing we do with the stack is pop the z off (as we always do) and push the z back on.
- We have a special state q_f of $\hat{M}$ that is not a state of M, and we create a transition $(f, \lambda, z) \to (q_f, \lambda)$ for every final state $f \in F$ of M.

(a) When talking about creating a context-free grammar from a pushdown automaton in class, we did not talk about what to do for transitions where the stack length stays the same (so we push one symbol back on the stack in a transition). Figure out the right thing to do for this case.

(b) If we start with a finite automaton M and create a pushdown automaton $\hat{M}$ as above, we can then create a context free grammar from $\hat{M}$. Describe all the USEFUL productions of this context free grammar. (Explain why the other productions are useless.) Compare this context free grammar to the construction of a right-linear grammar from a finite automaton M we did several weeks ago. What are the differences, if any?