Formal def'n of a DFA:

A DFA is a 5-tuple $M = (Q, \Gamma, \delta, q_0, F)$ where:

Q is a finite set (of states)

Γ is a finite set (the alphabet)

$\delta : Q \times \Gamma \longrightarrow Q$ is the transition function

$q_0 \in Q$ is the start state

$F \subseteq Q$ is the set of final states.

<u>Def'n</u>: Given a DFA M, define its generalized transition function

$$\delta_M^* : Q \times \underbrace{\Gamma^*}_{\text{set of strings on } \Gamma} \longrightarrow Q$$

by

$\xleftarrow{}$ empty string ('lambda)

0) $\delta^*_M(q, \lambda) = q$

1) Given a string $w = w'\ell$, $w \in \Gamma^*$, $\ell \in \Gamma$, $w' \in \Gamma^*$,

$$\delta^*_M(q, w) = \delta(\delta^*_M(q, w'), \ell).$$

<u>Def'n</u>: A DFA M <u>accepts</u> a string w if

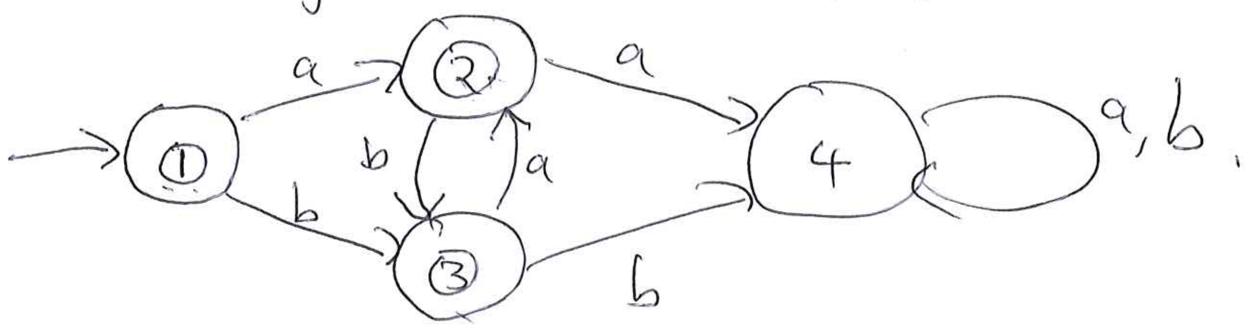
$$\underbrace{\delta^*(q_0, w)}_{\text{the result of applying the gen. trans. func to } w \text{ starting at } q_0} \in \underbrace{F}_{\text{the set of final states}}.$$

the result of applying the gen. trans. func to w starting at q_0

is in

the set of final states.

Working out the formalism:



$q_0 = 1$

$F = \{1, 2, 3\}$,

$\delta(1, a) = 2$, etc.

Let $w = aba$. Why is w accepted?

We have to calculate $\delta^*(q_0, w)$ and see if it's a final state.

$$
\begin{aligned}
\delta^*(1, aba) &= \delta(\delta^*(1, ab), a) \\
&= \delta(\delta(\delta^*(1, a), b), a) \\
&= \delta(\delta(\delta(\delta^*(1, \lambda), a), b), a) \\
&= \delta(\delta(\delta(1, a), b), a) \\
&= \delta(\delta(2, b), a) \\
&= \delta(3, a) \\
&= 2.
\end{aligned}
$$

Since $2 \in F$, M accepts w.

Proving facts about DFAs

Goal: write out a proof that our DFA accepts ~~precisely~~ the strings where a's + b's alternate and no others.

Pf: If $w = \lambda$, then $\delta^*(q_0, w) = \delta^*(1, \lambda) = 1$, and $1 \in F$, so $w = \lambda$ is accepted.

Claim: If w is an alternating string ending in a, then $\delta^*(1, w) = 2$. If w is an alternating string ending in b, then $\delta^*(1, w) = 3$. If w is not an alternating string, $\delta^*(1, w) = 4$.

Pf of claim: By induction on the length of w. Base case: $|w| = 1$. Then w is always alternating. If $w = a$, then $\delta^*(1, a) = \delta(\delta(1, \lambda), a) = \delta(1, a) = 2$. If $w = b$, then $\delta^*(1, b) = 3$. Both are as claimed.

Inductive case: $|w| > 1$. ~~Then~~ If w is alternating, then $w = w'\ell$, where w' is alternating, and $\ell \neq$ the last letter of w'. If the last letter of w' is an a, then by ~~the~~ induction, we can assume $\delta^*(1, w') = 2$. When w is alternating, in this case, $\ell = b$, and $\delta(2, b) = 3$, so $\delta^*(1, w) = \delta(\delta^*(1, w'), \ell)$ $= \delta(2, b) = 3$, as desired.

If, on the other hand, the last letter of w' is a b, then $\delta^*(1, w') = 3$, $\delta(3, a) = 2$, and $\delta^*(1, w) = 2$.

If w is not alternating, ~~either~~ $w = w'\ell$, where either w' is not alternating, or $\ell =$ last letter of w'.

If w' is not alternating, $\delta^*(1, w') = 4$ by induction, so $\delta^*(1, w) = \delta(4, \ell) = 4$

If $\ell =$ last letter of w', then if $\ell = a$, $\delta^*(1, w') = 2$ and $\delta(2, a) = 4$, so $\delta^*(1, w) = 4$
If $\ell = b$, ..., $\delta^*(1, w) = 4$.

This proves our claim.