Last time nondet fin automata.
-we have choice of which way to go in some situations
-if there is a path toward a final state, the string is accepted
-if no path to a final state, the string is rejected.

Reason for non-det automata - they are easier to construct.

Goal: Show how to take an NFA $M$ and construct a DFA $M'$ that accepts and rejects where $M$ and $M'$ accept and reject the same strings.
Easiest in an example:
(from last time)  

\[ 1 \xrightarrow{ab} 2 \]
\[ 1 \xrightarrow{aab} 3 \]

\[ b \]

\[ a \]

Idea: Keep track of the set of states we could be in.

Before we do this, it's easiest to work with single letter transitions—
change the NFA to an equivalent one:

(by sticking new states into the middle of arrows w/ long strings)
Clean-up picture
Write this in math:

Given an NFA $M = (Q, \Gamma, S, q_0, F)$, construct a DFA $\overline{M} = (\overline{Q}, \overline{\Gamma}, \overline{S}, \overline{q_0}, \overline{F})$ as follows:

$\overline{\Gamma} = \Gamma$

$\overline{Q} = \mathcal{P}(Q) = \left\{ \text{all subsets of } Q \right\}$

(In our example, we would have $2^b = 32$ states, but most of them were unreachable so we didn't bother drawing them.)

$\overline{F} = \left\{ S \in \mathcal{P}(Q) : S \cap \overline{F} \neq \emptyset \right\}$

$\overline{q_0} = \left\{ \text{all states in } Q \text{ reachable from } q_0 \text{ by } \lambda \text{-transitions} \right\}$

$\overline{S}$ is defined by
\[ S(q, l) = \bigcup_{q' \in \overline{q}} \{(q, l), q'\} \cup \{(q, l), q'\} \epsilon S \]

State of \( M \)-the DFA, which "is" a set of states of \( M \)-the NFA.

But, in \( S \), so far, I've forgotten about using \( \lambda \)'s to move other places after reading my letter.

Recall - \( S \) is a relation; \( ((q, l), q') \epsilon S \) means we have

\[
\xymatrix{ q \ar[r]^l & q' }
\]

in NFA.

Anytime you face the same problem twice, define a function to take care of it: Let

\[ E(q) = \{ \text{all states in } Q \text{ reachable from } q \text{ by } \lambda \text{-transitions} \} \]

Then

\[ \overline{q_0} = E(q) \]
$$\delta(q, l) = \bigcup_{q' \in \overline{q}} \bigcup_{q' \in \{q' \mid (q, l), q' \in S\}} E(q')$$
Another example

DFA: