Regular Expressions

<u>Def</u>: A language (i.e. a set of strings) is <u>regular</u> if it is possible to construct a $\boxed{\text{DFA}}$ that accepts the language (and rejects everything not in the language).

b/c of what we talked about last time, this is the same as constructing an NFA.

Note: 1) We can prove a language is regular by constructing a DFA (or NFA) for it

2) We don't yet (in the class) know ways to prove a language is not regular - an example of such a language is

$$\{a^n b^n \mid n \geq 0\} = \{\lambda, ab, aabb, aaabbb, aaaabbbb, \ldots\}$$

A regular expression is a way of describing a regular language by a single string.

Today and Friday:

1) Tell you what a regular expression is, and what language a reg. expr. stands for,
2) How we can construct a NFA that accepts the lang. described by a reg. exp.
3) How we can construct a reg. exp. ~~given an NFA.~~

that describes the lang. accepted by any given NFA.

~~A regu~~

<u>Def:</u> A <u>regular expression</u> on an alphabet Σ is a string on $\Sigma \cup \{\emptyset, +, (,), *, \lambda\}$ that can be formed using the following rules.

1) a) Any character in Σ is a reg. expr.
   b) λ is a reg. expr.
   c) $\emptyset$ is a reg expr.

2) Given any two reg. expr. u, v,
   a) $u+v$ is a reg. expr.
   b) (u) is a reg. expr.
   c) uv is a reg expr.
   d) u^* is a reg expr.

<u>E.g.</u> $a^*(b+ab)^*a+(ba)^*a$

is a reg. expr.

The language described by a regular expression is as follows:

~~1)~~ E is the expression, L(E) is lang. (Terminology: a string w <u>matches</u> a reg. expr. E $\Leftrightarrow$ $w \in L(E)$)

1) a) $E = \ell$, $L(E) = \{\ell\}$.

b) $E = \lambda$, $L(E) = \{\lambda\}$

c) $E = \emptyset$, $L(E) = \emptyset$

2) a) $u+v$: $L(u+v) = L(u) \cup L(v)$.

(i.e. A string w matches $u+v$ if w matches u or w matches v)

b) (u) $L((u)) = L(u)$

c) uv: $L(uv) = L(u)L(v)$

(A string w matches uv if there are strings w', w'' where $w = w'w''$, and w' matches u, w'' matches v)

d) u^* $L(u^*) = L(u)^*$

A string w matches u^* if ~~there~~ either $w = \lambda$, or

$w = w_1 \cdots\cdots w_k$ (for some k), where w_i matches u for all i.

E.g. a^* means any number of a's (i.e. a, aa, aaa, ... all match a^*)

E.g. $(ba)^*$ means any number of ba's (i.e. ba, $baba$, $bababa$, ---- all match ba^*)

E.g. $(b+ab)^*$ is matched by λ, b, ab, bb, bbb, $b|ab|ab|b|ab$

Strings matching

$$\underbrace{a^*(b+ab)^*a}_{} + (ba)^* a$$

and ending ~~at least~~ with ~~one~~ a

?

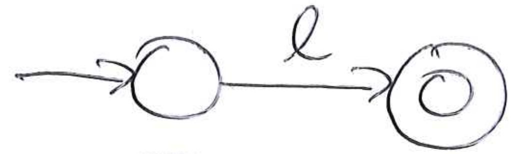
any string w/no consec a's except at beginning?

babaa, aaababbbbaba

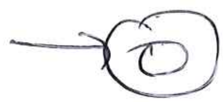
Next goal: given a regular expr., build an NFA that accepts the strings matching the reg. exp. (and no others)

We do this by going through the possible methods for building a reg expr and see how to build a ~~mach~~ corresp. NFA.

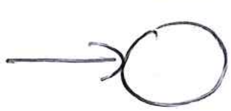
1) a) $E = e$



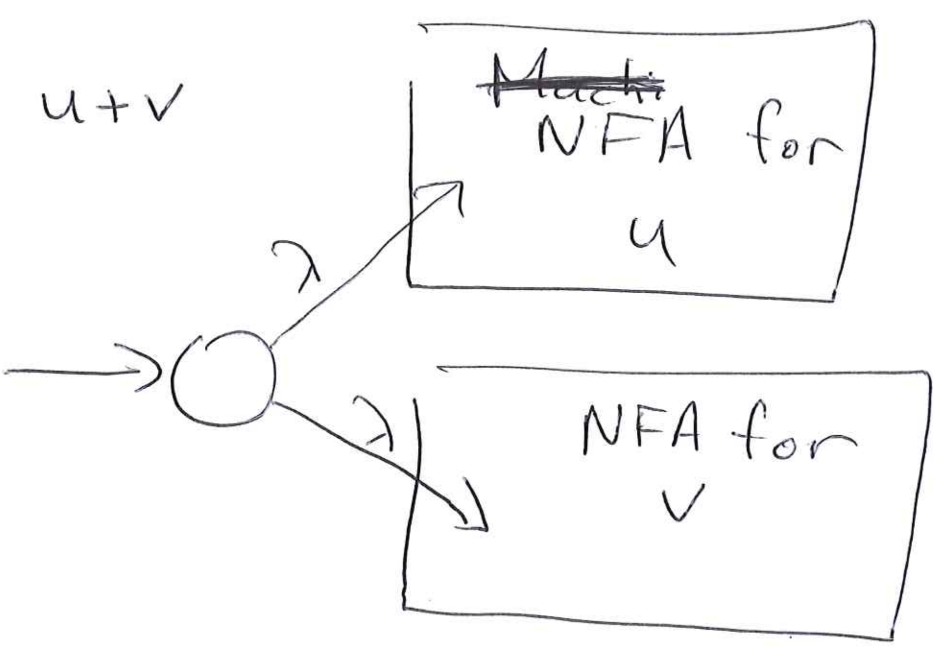
b) $E = \lambda$



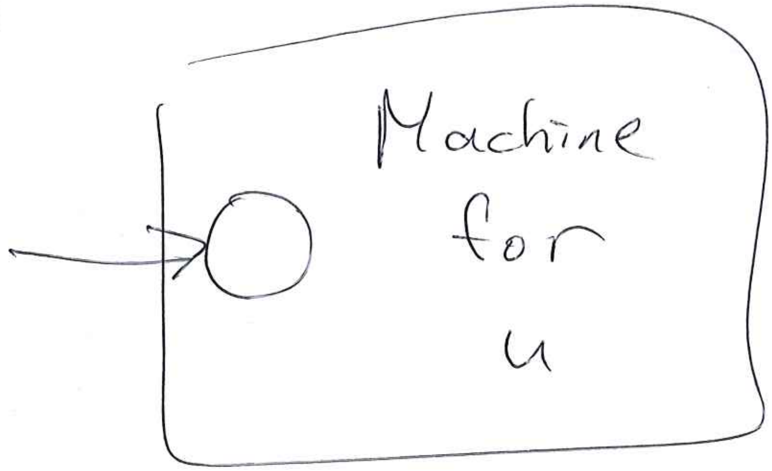
c) $E = \emptyset$



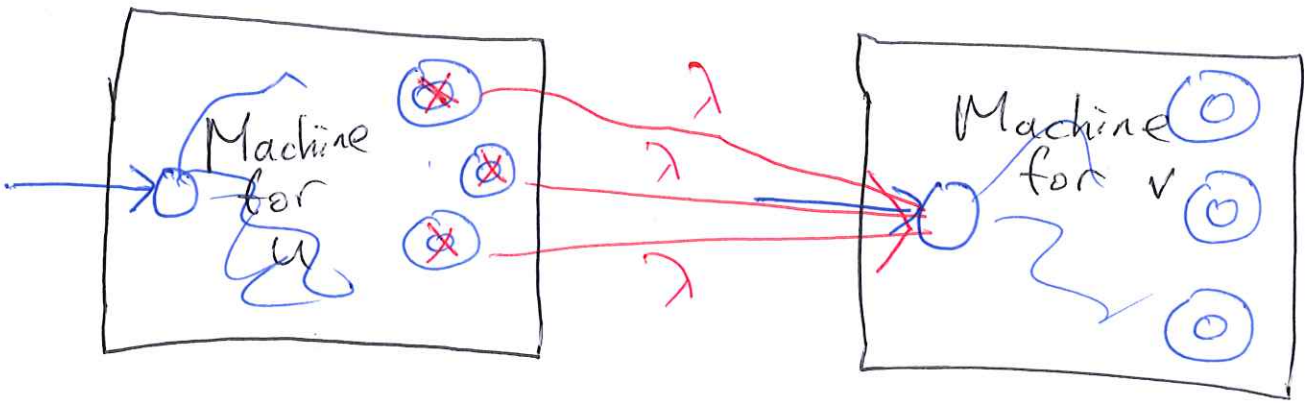
2) a) $u + v$



b) (u)



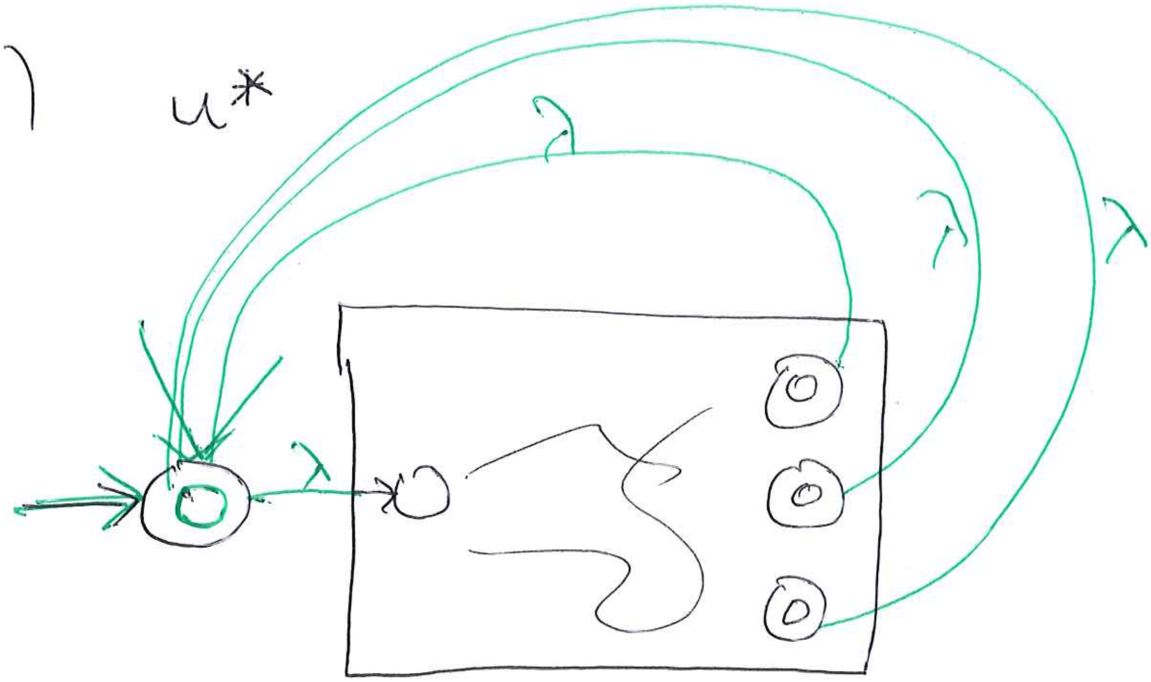
c) uv



Connect all the final states of NFA for u to the initial state of NFA for v.

The final states of the new machine are final states in NFA for v (but final states in NFA for u are not final)

d) u^*

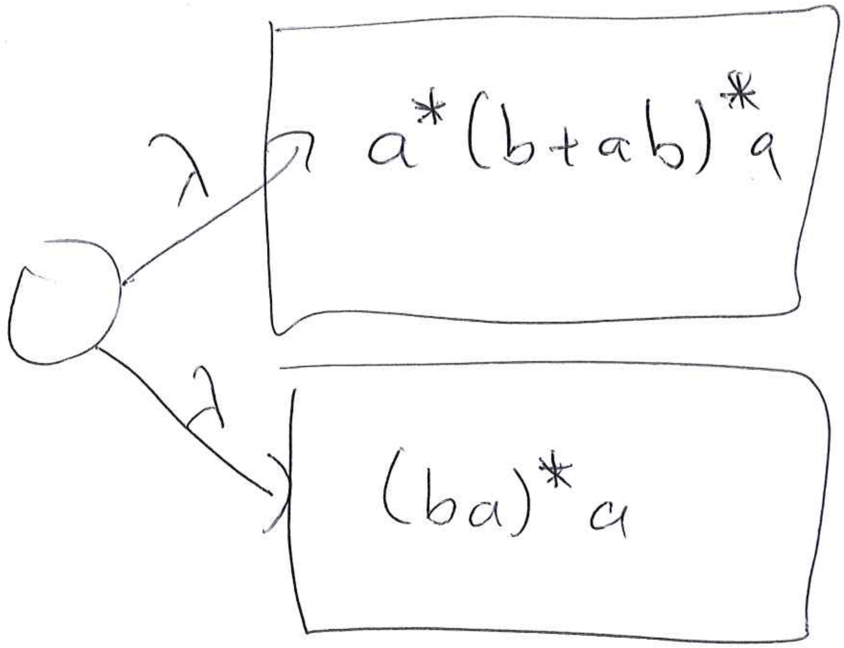


NFA for u

(Make a single new state, which is final, w/ a λ-transition to the init state of u and λ-transitions from all the final states of u)

E.g. $a^*(b+ab)^*a + (ba)^*a$

has the NFA



or