Regular Expressions

Last time:

1) What a reg expr is
2) What set of strings a reg expr stands for (aka what strings match a reg expr)
3) How to construct an NFA that accepts precisely the strings that match a given reg. expr.

Today: How to write a reg. expr. for the set of strings accepted by an NFA.

Review example

\((a^*ba^*(a+b))^*a\) is a reg expr.

Construct an NFA that accepts this reg. expr.:
simplify this:

No good—this would let me repeat a's and then use an a to go to final state.
What about $\emptyset$ in reg. expr.?

$((a+\emptyset)a^*(b+\emptyset))^* \equiv aa^*b$

(since "a+\emptyset" and "a" are both matched only by a)

$a\emptyset \equiv \emptyset$ (since nothing can be split up into and a and ...uh...)

(but

$a\lambda \equiv a$

Actually, the $\emptyset$ is mostly useless; anytime it is combined w/something you can simplify the result to either just $\emptyset$ or something w/o $\emptyset$. The only use is to specify the reg expr that matches nothing.
Turning NFAs into reg. exprs:

Define a "super-NFA" (the book calls them "generalized transition graphs") to be a NFA where the arrows are labelled with reg. expr.

(To use a path, you pay any toll that matches the reg. expr.)

E.g.

\[
(a+b)^*a [(a+ab) + (b^*b(a+b))^* (a+b)^*] 
\]
If I have just 2 states, 1 of which is final, I can figure out the reg. expr.

For a general super-NFA, I can
0) convert to have only one final state.
1) reduce the number of states by making my transitions more complicated:

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E.g.

\[
1 \rightarrow a \rightarrow 2 \\
\text{(b+a)*b} \quad \text{b} \\
\text{(a*ab+b)} \quad \text{(b*ab)} \\
2 \rightarrow a \rightarrow \quad \text{(a+b)*} \\
\text{3} \quad \text{b} \\
\text{(a+ab)}
\]
If I want to get rid of $\mathbf{3}$, I need to add extra possibilities to my transitions to account for the transitions that used $\mathbf{3}$.

To get from 1 to 2, I can either go directly or go loop around 3, and then go 3→2.
2) Keep reducing the number of states, getting rid of all states except the initial state and the final state.

3) With the 2 state super-NFA, write down the reg. expr.