

$\text{truncate}(\text{ababba}) = \text{ababb}$

~~If  $w$~~

set of strings

$$\text{truncate}(L) = \{\text{truncate}(w) : w \in L\}$$



We're overloading this function name

result is a set of strings

$\text{truncate}(\{\text{ababba}, \text{bab}, \text{aabab}\})$

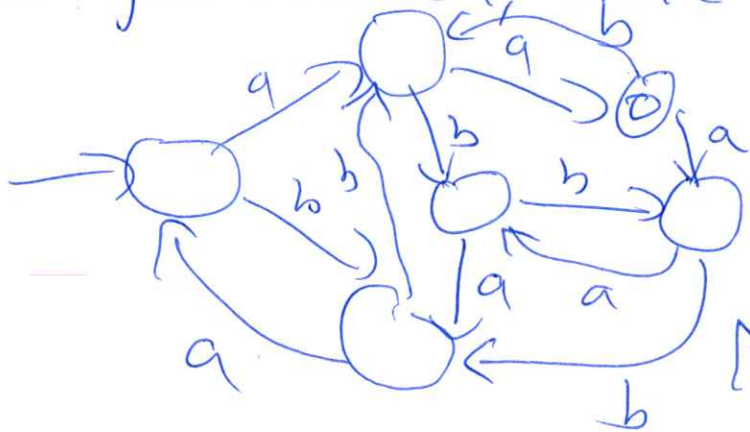
$= \{\text{ababb}, \text{ba}, \text{aaba}\}$

Given some unknown DFA ~~you are~~ for  $L$   
~~given~~ construct a DFA for  $\text{truncate}(L)$ .

This means - describe a computer program that takes as input a DFA  $M$ , and outputs a DFA  $M'$ , so that if  $L$  is the language accepted by  $M$ , then  $M'$  accepts  $\text{truncate}(L)$ .

To do this problem, you need to be specific - someone should be able to write the code from your description.

To get started, make a random machine



and think

about what machine would accept  $\text{truncate}(L)$  (where  $L$  is the language accepted by )

# Regular grammars

So far, we have been thinking about ~~the~~ regular languages - by def'n a ~~the~~ set of strings that is the set accepted by some DFA.

We have 2 ways so far to describe a regular lang -

- give a DFA <sup>or NFA</sup> for it

or - give a reg. exp. for it.

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Today: Another way to describe languages in general (and specifically regular lang.)

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A grammar a set of rules for generating a language that has a specific form (later)

E.g.  $\Gamma = \{a, b\}$   $V = \{S, C, D\}$

$$S \rightarrow aCb$$

$$S \rightarrow CbD$$

$$C \rightarrow CD$$

$$C \rightarrow abD$$

$$C \rightarrow b$$

$$D \rightarrow DCSa$$

$$D \rightarrow a$$

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To generate strings using a grammar, I start with the "start symbol"  $S$ , and keep making substitutions until all my symbols are in  $T$ .

$$\begin{aligned} S &\rightarrow CbD \rightarrow CDbD \rightarrow bDbD \\ &\rightarrow bDCSbD \rightarrow baCSabD \\ &\rightarrow babSabD \rightarrow babaCbabD \\ &\rightarrow \boxed{bababbaba} \end{aligned}$$

The final result is "a string generated by the grammar"

The language generated by a grammar is the set of strings that can be generated by the grammar.

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Formal definition:

A grammar is a set  $(V, \Gamma, P, S)$  where

$V =$  finite set of symbols (variables)

$\Gamma =$  finite ~~set~~ of alphabet (terminal symbols)

(We assume  $V \cap \Gamma = \emptyset$ )

$S \in V$  the start symbol

$P \subseteq \cancel{V}^* \times (V \cup \Gamma)^*$  are my production rules.

$V^* - \{\lambda\}$

↑  
not allowed to replace nothing w/something.

Def: A string  $w$  is produced by a grammar

$G = (V, T, P, S)$  if

a)  $w \in T^*$

1) There is some sequence of strings

$v_0, \dots, v_k \in (V \cup T)^*$  where

$v_0 = S$

$v_k = w$  for each  $i$ ,

and "I can get from  $v_i$  to  $v_{i+1}$  by following a rule in  $P$ "

$v_i = xyz, v_{i+1} = xy'z$ , where

$x, y, y', z \in (V \cup T)^*$ , and  $(y, y') \in P$



This is the formal way to say  
"replace  $y$  by  $y'$  to get from  
 $v_i$  to  $v_{i+1}$ "

Some boring grammars: (conventions: capital letters are in  $V$  lower case in  $T$ )

$$S \rightarrow \cancel{aTb} aSb$$

$$S \rightarrow \lambda$$

generates a's followed by b's, with same number of each.

$$S \rightarrow \cancel{aTb} abT$$

$$\cancel{S \rightarrow \lambda}$$

$$S \rightarrow T$$

$$S \rightarrow bT$$

$$T \rightarrow abT$$

$$T \rightarrow \lambda$$

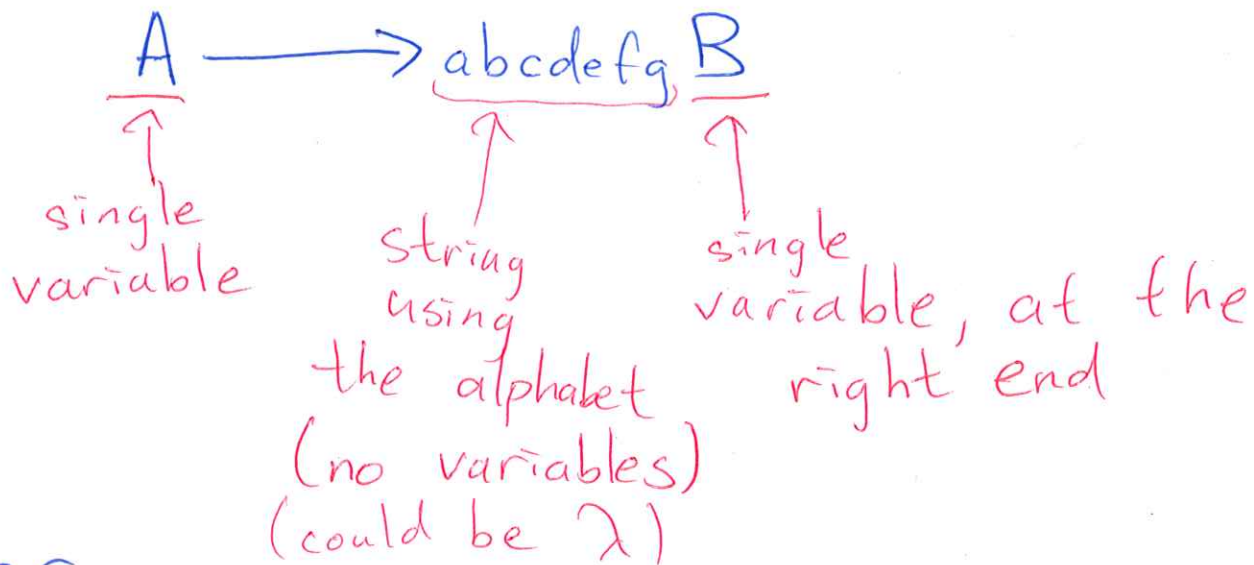
bab, babab, bababab  
 $\lambda$ , ab, abab, ...

strings that alternated between a's + b's that end with a b.

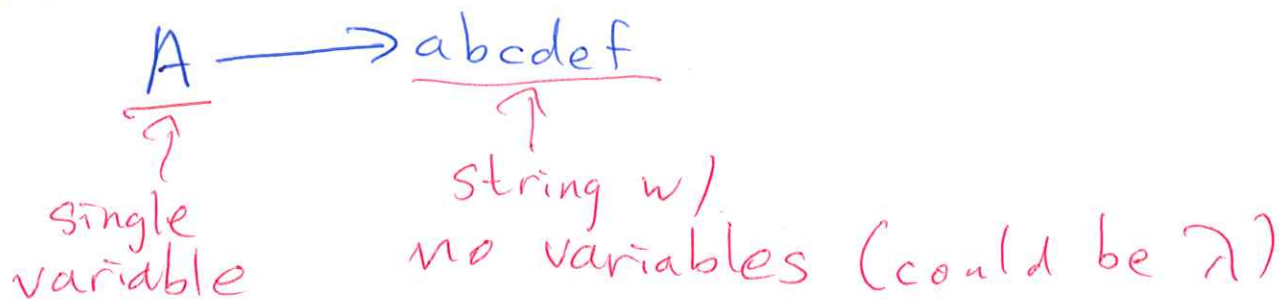
add  $T \rightarrow a$   
 or add  $S \rightarrow Ta$

to allow strings to also end with a.

Def: A grammar is right-linear if every production in  $P$  looks like



or



Next time - explain the corresp. between right-linear grammars and NFAs.