Grade interpretation on Exam 1

55 - 70 C

70 - 85 B for this exam

85 - 100 A

Today: More about using the pumping lemma to prove that languages are not regular.

Pumping Lemma: Let L be a regular language. Then there exists a positive integer m so that, for every string $w \in L$ with $|w| > m$, there exist ~~x, y, z~~ strings x, y, z, $y \neq \lambda$, $|xy| \leq m$ such that $w = xyz$ and ~~wy~~ $xy^i z \in L$ for all nonnegative integers i.

To show a language is not regular, you can show that the conclusion of the pumping lemma is false.

- You don't get to choose m - you have to handle all possibilities
- For each m, you choose a $w \in L$, that you will use to "break" the pumping lemma.
- Given w, you don't get to choose x, y, z; you have to handle all possibilities ~~for~~ (w/ $w = xyz$, $y \neq \lambda$, $|xy| \leq m$) You probably want to
- have chosen w so that there aren't too many possibilities.
- ~~Given~~ For each possibility for x, y, z, you choose an i so that

  $xy^iz \notin L$.

E.g. $L = \{a^k b^n \mid k \geq n\}$

Claim: L is not regular. $w = a \cdots a b \cdots b$

Pf: Given m, let $w = a^m b^m \in L$. Suppose $w = xyz$, $|xy| \leq m$, $y \neq \lambda$. Then $y = a^j$ for some j, $1 \leq j \leq m$. ~~Then~~ In this case, $xy^0z = xz = a^{m-j} b^m$. Since $m - j < m$ (as $j \geq 1$), $a^{m-j} b^m \notin L$. Hence the pumping lemma fails, so L is not regular.

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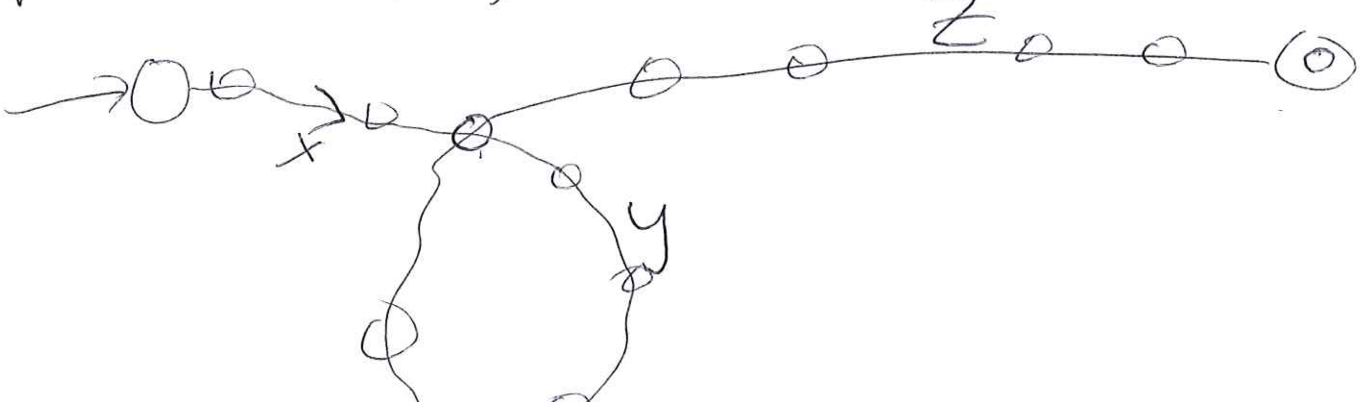
path for accepting w



E.g. Let $L = \{ a^{n^2} \mid n \in \mathbb{Z} \}$

$$L = \{ \lambda, a, aaaa, \underbrace{a \cdots a}_{9}, \underbrace{a \cdots a}_{16}, \ldots \}$$

Claim: L is not regular.

Pf: Given m, let $w = a^{m^2} \in L$. ~~Then~~
Suppose $w = xyz$, $y \neq \lambda$, $|xy| \leq m$. Then $y = a^j$, where $1 \leq j \leq m$. Hence

$$xy^2z = a^{m^2+j}. \quad \text{But}$$

$$m^2 < m^2+j \leq m^2 + m \leq m^2 + 2m + 1 = (m+1)^2$$

(since $m+1 > 0$), so $m^2 + j \neq n^2$ for any $n \in \mathbb{Z}$, and $a^{m^2+j} \notin L$. Hence the pumping lemma fails for L and L is not regular.

E.g. $L = \{ w \in \{a,b\}^* \mid \#a\text{'s} = \#b\text{'s} \}$.

If we use pumping lemma:

Given m, pick $w = a^m b^m$

If $w = xyz$ $y = a^j$ - delete y gives $a^{m-j} b^m \notin L$.

We used the same string as for

$L'' = \{a^m b^m\}$.

Another way to prove L is not regular.

Pf: Let $L' = \{a^m b^m\}$. We know

$L' = L \cap$ the lang of the reg exp $a^* b^*$.

We know L' is not regular. (We proved this on Monday.) If L were regular, then L' would be the intersection of 2 regular languages and hence regular. This is a contradiction, so L can't be regular.