Last time: given a CFG, how to make a PDA that accepts the lang. gen. by the CFG

Key idea: Modify the CFG so that, in a leftmost deriv, we always have letters vars. Then have the stack of the PDA keep the vars.

Today: Given a PDA, how to make a CFG that generates the lang accepted by the PDA.

Key idea: Variables in CFG will look like

[state, stack symbol, state] - this variable can generate any string that is accepted by the PDA going from left state to right state consuming the stack symbol from stack.

↑
one variable

Given a PDA - we'll make ~~it~~ an equivalent PDA so that

① it has one final state, and when you enter this final state, there is nothing on the stack.

② we'll assume for ease of description that every transition either decreases stack size by one, or increases stack size by one.

The start ~~for~~ variable for our CFG is $[q_0, Z, q_f]$ (where q_0 = start state
Z = bottom of stack symbol
q_f = final state)

Given a transition

$$(q, \ell, s) \longrightarrow (q', \lambda),$$

we have a production

$$[q_0, s, q'] \longrightarrow \ell$$

$(q, \ell, s) \longrightarrow (q', tu)$

we have ~~transitions~~ productions

$[q, s, p] \longrightarrow \ell [q', t, p'] [p', u, p]$

for every ~~p~~ state p' and every state p.

Note - this generates n^2 ~~t~~ productions, where n = # states in PDA.

Most of these productions will be useless - but we know how to get rid of useless productions and can do that afterwards.

Example: The PDA that accepts ~~$\{a^n b^n\}$~~ $\{a^n b^n \mid n \geq 1\}$

It's transitions are

$(q_0, a, z) \longrightarrow (q_1, Az)$

$(q_1, a, A) \longrightarrow (q_1, AA)$

$(q_1, b, A) \longrightarrow (q_2, \lambda)$

$(q_2, b, A) \longrightarrow (q_2, \lambda)$

$(q_2, \lambda, z) \longrightarrow (q_f, \lambda)$

The CFG has $4 \times 2 \times 4 = 32$ variables

It's productions are:

$[q_1, A, q_2] \rightarrow b$

$[q_2, A, q_2] \rightarrow b$

$[q_2, Z, q_f] \rightarrow \lambda$

$[q_0, Z, \_] \rightarrow a\ [q_1, A, \_]\ [\_, Z, \_]$

$[q_0, Z, q_0] \rightarrow a\ [q_1, A, q_0]\ [q_0, Z, q_0]$

$[q_0, Z, q_0] \rightarrow a\ [q_1, A, q_1]\ [q_1, Z, q_0]$

$[q_0, Z, q_0] \rightarrow a\ [q_1, A, q_2]\ [q_2, Z, q_0]$

$[q_0, Z, q_0] \rightarrow a\ [q_1, A, q_f]\ [q_f, Z, q_0]$

$= q_0$

$[q_0, Z, q_1] \rightarrow a\ [q_1, A, q_0]\ [q_0, Z, q_1]$

$[q_0, Z, q_1] \rightarrow a\ [q_1, A, q_1]\ [q_1, Z, q_1]$

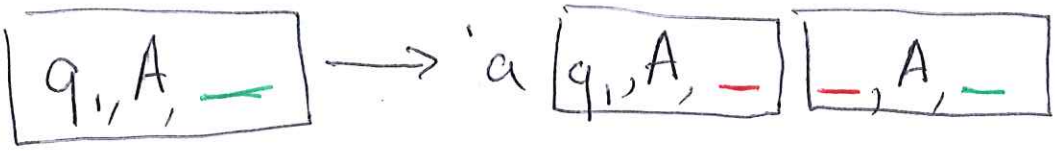
$[q_0, Z, q_1] \rightarrow a\ [q_1, A, q_2]\ [q_2, Z, q_1]$

$[q_0, Z, q_1] \rightarrow a\ [q_1, A, q_f]\ [q_f, Z, q_1]$

$= q_1$

$\vdots$

+ 8 more



16 of these

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actually, most of these are useless.