

# The universal Turing machine

Idea: specific  $TM^U$  that takes in a  $TM^M$  as input, along with input  $I$  to ~~that TM~~  $M$ , and do whatever  $M$  does on  $I$ .

(i.e.  $U$  is a TM-interpreter)

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In order for this to make sense, we need some standard for

- 1) How to specify  $M$  (for  $U$ ) ~~so that~~
- 2) How to specify  $I$ .

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E.g. one possibility:

(I need to specify transitions, start state, final state)

- Note - an arbitrary TM has an arbitrary alphabet - I can't allow this for  $U$  since  $U$  must have a single

How can we construct  $U$ ?

We'll construct a 3-head 3-tape machine:

Tape 1: Always just has (the encoding of)  $M$ ,  
(well, there might be temporary changes)

Tape 2: The tape contents of  $M$  at the current point of the computation.  
(encoded (as 1's + #'s))

Tape 3: The current state of  $M$ .  
(encoded as some number of 1's)

~~(Maybe we need a tape~~  
(Maybe the current head position of tape 2 also has a special mark.)

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We have to make  $U$  simulate  $M$  - so we should think about the "loop" on  $U$  that simulates a single transition in  $M$ .

- 2) Update ~~the~~ tape 2 & tape 3
- copy the new state to tape 3  
(and blank the rest of it)
  - copy the new symbol to tape 2  
(moving the stuff to the right  
either to make room or to delete  
blank room as necessary)
  - move the marker for current  
spot on tape 2 as necessary.
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This is the loop for 1 step.