Plan for the rest of semester:

Today: Halting problem

Friday: ~~Equivalence o~~ languages gen by unrestricted grammars are TA and, every TA lang has a unrestricted grammar.

Next week: review

Halting problem: We have a way of encoding TM, and corresponding to that, encoding the input to our TM (we discussed this for the universal TM.) We have the language

$L = \{ w \in \{0,1\}^*$ where

w is an encoding of a TM M followed by an encoding of input I to that TM where M halts on input I $\}$

Thm: L is not TD.

Note: L is TA/TE - ~~just run~~ it is accepted by the universal Turing machine (modified to first check if we have a valid encoding, and also modified so that every halting state is an accept state).

Short but uninformative proof:

Suppose L were TD for contradiction. Then we will prove that every TA/TE language is TD. How?

If we have a TA lang. X, there is a TM M that accepts X. ~~To~~ We construct a TM $\hat{M}$ that decides X. $\hat{M}$ first feeds M and w (w is our candidate for being in X) to the TM that decides L.

If this TM says M doesn't halt on w, $\hat{M}$ should say no.

If this TM says M does halt on w, $\hat{M}$ should run M and return M's answer.

No matter what, $\hat{M}$ halts, and it always gives the right answer.

So $\hat{M}$ decides X, so X is TD.

We know that not every TA lang is TD, so our assumption that L is TD must be false.

Longer, more informative proof:

Assume for contradiction that L is TD.
Then there is a TM H that decides L.

Method: We will construct from H a TM that has paradoxical behavior - ~~it will~~ there will be a string which it should both accept and reject (should neither reject nor accept).

0) From H, construct H' that will halt only in one of two states - one final state q_f and one non-final state q_n.

1) From H' construct H'' so that,
   a) When H' enters q_f, H'' goes into an infinite loop. (add transitions $(q_f, \text{anything}) \to (q_f, \text{same thing}, R)$)
   b) The behavior on entering q_n is untouched.

H'' does the following:

- it's input is a TM M, followed by input to M (called w) - all suitably encoded.
- if M halts on w, then H'' runs forever
- if M doesn't halt on w, then H'' halts. (in q_n)

2) Construct a TM $\hat{H}$ that first copies its input, ~~and~~ (so if it starts with input w, ~~it ha~~ puts $w\,O\,w$ on the tape), and moves the head all the way back to the beginning, then runs H''.

$\hat{H}$ is a TM, so we can encode $\hat{H}$ as a string and feed it as input to $\hat{H}$.

What happens?

$\hat{H}$ on any input should either run forever, or halt in state q_n.

Suppose $\hat{H}$ runs forever, given the string for $\hat{H}$ as input. Then $\hat{H}$ actually does the following:

It duplicates the input to $\hat{H} \circ \hat{H}$ (as strings) and feeds answer to H''. Since ~~H~~ $\hat{H}$ doesn't halt on the input $\hat{H}$ (as a string), H'' halts in q_n.

Suppose $\hat{H}$ halts given $\hat{H}$ as input, then $\hat{H}$ feeds $\hat{H} \circ \hat{H}$ to H'', and runs forever.

This is nonsense, so $\hat{H}$ can't exist, so H'' and H' and H can't exist.