- Turing machine w/ a stay option.
  (i.e. the transition function now is
  \[ S : Q \times \Gamma \rightarrow Q \times \Gamma \times \{ L, R, S \} \]
  (w/ the relevant def'n of how instants
descr. are affected by \( S \))

- Turing machines w/multiple tapes (one head)
- Turing machines w/multiple heads

  - there is a complicated way to simulate w/ a standard TM.

Nondet-TM

\[
\begin{align*}
(q_0, \square) & \rightarrow (q_0, \square, L) \\
(q_0, \square) & \rightarrow (q_0, \square, R)
\end{align*}
\]
(The stupidest & non-det TM possible)

Def: A non-det TM accepts a string \( w \) if any of the \( \omega \) sequences of
transitions allowed leads to a halting in
a final state.
Construct a TM that multiplies its input by 3.

E.g. 90

And

011100

11yy

10yy

xx1yy y y y y

xxx y y y y y y

change to all 1's.
Closure properties of CF langs

If $L_1 \cup L_2$ are CF langs:

$L_1 \cup L_2$ is CF

$L_1 L_2$ is CF

$L_1^*$ is CF

$L_1 \cap L_2$ is not in general CF
(but $L_1 \cap L_2$ is CF if $L_1$ is CF
$L_2$ is reg)

E.g. $L_1 = \{a^n b^n c^m | \text{any } n, m \}$
$L_2 = \{a^m b^n c^n | \text{any } n, m \}$

$L_1$ is not in general CF
Problem \( P \)

Input: TMs \( M_e + N \)

Output: \( Y \) if \( L(M) = L(N) \), \( N \) otherwise

Why is this undecidable?

Assume there is a TM \( P \) that solves \( P \).

Use \( P \) to solve Halting Problem.

Input A TM

\( w \) string

1. Modify A so that it accepts whenever it halts (by making every state final or changing all halting transitions to go to a special final state)
② Take A and modify it so that

a) First check if the input string is w.

b) If no, reject; if yes, run A (on w) no matter what input accepting on halting is.

So A_w - if A halts on w,

\[ L(A_w) = \{w\} \text{ every string} \]

If A doesn't halt on w,

\[ L(A_w) = \emptyset. \]

③ Make a TM \( W \) that accepts only w, every string.

④ Feed \( W \) and A_w to P.