Mid-term 2

October 25, 2018

(1) (20 points) Construct a pushdown automaton that accepts the language

$$\{a^k b^m c^n \mid k + m = n, k \geq 1, m \geq 1\}.$$

(2) Consider the grammar with variables $\{S, T, U\}$ on the alphabet $\{a, b\}$ with the following productions:

$$S \to aT|TU$$
$$T \to aTa|bTb|a|b|\lambda$$
$$U \to aUb|\lambda$$

(a) (8 points) Find a derivation (or a derivation tree) for the string $abaab$
(b) (10 points) Show that this grammar is ambiguous

(3) (12 points) Give the definition of a pushdown automaton.

(4) (12 points) Consider the grammar with variables $\{S, T\}$ on the alphabet $\{a, b\}$ with the following productions:

$$S \to ab|bST$$
$$T \to aT|bS|\lambda$$

Give an equivalent grammar in Chomsky Normal Form.

(5) (16 points) Consider the grammar with variable $\{S, B\}$ and productions

$$S \to aSB|SS|\lambda, \quad B \to b$$

If one follows the construction of a pushdown automaton from a grammar, one gets the pushdown automaton with 3 states q_0, q, q_f and transitions

$$(q_0, \lambda, Z) \to (q, SZ)$$
$$(q, \lambda, S) \to (q, \lambda)$$
$$(q, \lambda, S) \to (q, SS)$$
$$(q, a, S) \to (q, SB)$$
$$(q, b, B) \to (q, \lambda)$$
$$(q, \lambda, Z) \to (q_f, \lambda)$$

One way for this pushdown automaton to accept the string $aabb$ is as follows:

State	Remaining Input	Stack
q_0	$aabb$	Z
q	$aabb$	SZ
q	$aabb$	SSZ
q	$aabb$	SZ
q	abb	SBZ
q	abb	$SSBZ$
q	bb	$SBSBZ$
q	bb	$BSBZ$
q	b	SBZ
q	b	BZ
q	λ	Z
q_f	λ	λ

Give the leftmost derivation of the string $aabb$ in the grammar that corresponds to this path through the pushdown automaton. (Note that this grammar is ambiguous, so there are many ways to derive this string, but only one of those corresponds to this particular path.)

(6) (20 points) Show that

$$\{a^n b^k \mid k \leq n \leq k^2\}$$

is not a regular language.