Final Exam

December 11, 2018

(1) (10 points) Consider the Turing machine with initial state q_0 and transitions

$$\begin{aligned}
\delta(q_0, a) &= (q_1, x, R) \\
\delta(q_0, b) &= (q_2, y, R) \\
\delta(q_0, x) &= (q_2, y, R) \\
\delta(q_0, y) &= \mathcal{H} \\
\delta(q_1, a) &= (q_1, y, L) \\
\delta(q_1, x) &= (q_2, y, R) \\
\delta(q_1, b) &= (q_0, b, L) \\
\delta(q_1, y) &= \mathcal{H} \\
\delta(q_2, a) &= (q_2, b, L) \\
\delta(q_2, b) &= (q_1, a, L) \\
\delta(q_2, y) &= (q_0, a, R) \\
\delta(q_2, x) &= \mathcal{H}
\end{aligned}$$

(In addition, the Turing machine halts anytime a blank is read.)

Suppose this Turing machine is started with the string ab on the tape, with the head over the a. Give the sequence of instantaneous descriptions as this Turing machine runs.

(2) (25 points) Construct a Turing machine that multiplies its input (thought of as a unary number) by 3. Explicitly give all the transitions.

(3) (15 points) Give definitions stating what it means for a language to be Turing-decidable and Turing-acceptable. Explain why all Turing-decidable languages are Turing-acceptable. Give an example (no proof necessary) of a language that we know is Turing-acceptable but not Turing-decidable.

(4) (25 points) Explain why there cannot be a Turing machine that takes as input two (encodings of) Turing machines and decides if there is a string accepted by both of them. (You may use without proof the fact that the usual halting problem is undecidable.)

(5) (25 points) Sketch the operation of the universal Turing machine (as a multihead, multitape Turing machine).

(6) (25 points) Show that

$$L = \{a^j b^k c^n \mid j < k < n\}$$

is not context free.

(7) (15 points) Give a context-free grammar that generates the language L of strings on $\{a, b\}$ where the number of a's is exactly 2 more than the number of b's.

(8) (10 points) Find a derivation tree for $abbabb$ given the grammar $S \to SS|aSa|bSb|\lambda$.

(9) (15 points) Show that

$$L = \{a^k b^m c^n \mid k = m + n\}$$

is not regular. You may use the fact that the language

$$L' = \{a^n b^n \mid n \geq 0\}$$

is not regular.

(10) (10 points) Construct an NFA that accepts precisely the strings generated by the grammar with variables $V = \{S, A, B\}$ and productions

$$S \to aS|aA, A \to bB|S|bB, B \to cA|a$$

(11) (10 points) Construct a DFA that accepts precisely the strings on $\{a, b\}$ with an even number of a's and an odd number of b's.

(12) (15 points) Give the formal definition of a deterministic finite automaton. Be sure to state all the restrictions on the sets and functions that make up the automaton. (I want just the definition of the data structure and don't need anything about how it operates.)