Single and Three Phase Systems Summary

1. Notation for instantaneous and phasor quantities:
   
   (a) Instantaneous: \( v(t) = V_{\text{max}} \cos(\omega t + \theta_v) \)
   
   (b) RMS Phasor quantity: \( V = \frac{V_{\text{max}}}{\sqrt{2}} / \theta_v \)

2. Instantaneous single phase power: \( p(t) = \frac{1}{2} V_{\text{max}} I_{\text{max}} [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)] \)

3. Average single phase power: \( P_{\text{av}} = \frac{1}{2} V_{\text{max}} I_{\text{max}} [\cos(\theta_v - \theta_i)] \)

4. Power Factor
   
   (a) Power factor angle (\( \phi \)) = \( \theta_v - \theta_i \)
   
   (b) \(-90^\circ \leq \phi \leq 90^\circ\)
   
   (c) Power factor: \( p_f = \cos(\phi), 0 \leq p_f \leq 1 \)
   
   (d) Lagging Power Factor: \( \phi > 0 \)
   
   (e) Leading Power Factor: \( \phi < 0 \)

5. Three Phase Power
   
   (a) \( S_3 = 3V_0 I_0^* \)
   
   \( |S_3| = \sqrt{3} |V_0| |I_0^*| \)
   
   (b) \( P = 3|V_0| |I_0^*| \cos \phi \)
   
   (c) \( Q = 3|V_0| |I_0^*| \sin \phi \)
   
   (d) \( P = \sqrt{3} |V_0| |I_0^*| \cos \phi \) (\( \phi \) is the angle between Phase quantities!)
   
   (e) \( Q = \sqrt{3} |V_0| |I_0^*| \sin \phi \)
   
   (f) Power dissipated in shunt resistor (Y connected):
   
   \( P = \frac{3|V_0|^2}{R} = \frac{|V_0|^2}{R} \)
   
   (g) Reactive power to/from Y connected shunt reactance (capacitor or reactor)
   
   \( Q = \frac{3|V_0|^2}{X} = \frac{|V_0|^2}{X} \)

6. \( Z_\Delta = 3Z_Y \)

\( \text{inductive} \)
Transformers

1. Single Phase Ideal Transformer Relations:

\[ \frac{V_1}{N_1} = \frac{V_2}{N_2} = \frac{V_3}{N_3} \quad 0 = I_1 N_1 + I_2 N_2 + I_3 N_3 \]

2. Three Phase Transformers:

\[ \overline{V}_2 = K \overline{V}_1 \quad \overline{I}_2 = \frac{1}{K} \overline{I}_1 \]

\[ Y - Y : K_{YY} = \frac{N_2}{N_1} \]

\[ \Delta - \Delta : K_{\Delta\Delta} = \frac{N_2}{N_1} \]

\[ \Delta - Y : K_{\Delta Y} = \sqrt{3} e^{\pm j30^\circ} \frac{N_2}{N_1} \]

\[ Y - \Delta : K_{Y\Delta} = \frac{e^{\pm j30^\circ}}{\sqrt{3}} \frac{N_2}{N_1} \]

NOTE: The high voltage side leads the low voltage side by 30° for \( Y - \Delta \) and \( \Delta - Y \) transformers when compare Line-to-Line voltage to Line-to-Line voltage.
Per Unit

\[$ S_B \rightarrow 3\phi \]$ system wide

Voltage Bases

\[ V_{LLB} \] - define
\[ V_{LNB} = \frac{V_{LLB}}{\sqrt{3}} \]

Choose 1 as starting point

\( \rightarrow \) every time cross a transformer define a new base \( \rightarrow \)

\[ V_{B2} = V_{B1} \cdot \left( \frac{V_{T2}}{V_{T1}} \right) \] (Trans Voltage Trans Ratio)
$$I_{B1} = \frac{5B}{(5B + V_{B1})^2}$$

$$E_{B1} = \frac{V_{B1}}{5B}$$
To go back to ohms

1. RC, JXm are on LV
   - HV side is Δ

   ZB2 on LV side
   ZB3 on HV side
\[ R_{\text{i}}(\Omega) = R_{\text{i}} \cdot Z_{B2} \]

\[ X_{\text{i}}(\Omega) = X_{\text{i}} \cdot Z_{B2} \]

Base on LV side of transformer

\[ R_c(\Omega) = R_{\text{c}} \cdot Z_{B2} \]

\[ X_m(\Omega) = X_{\text{m}} \cdot Z_{B2} \]

\[ R_{\text{z}}(\Omega) = R_{\text{z}} \cdot Z_{B3} \cdot 3 \]

\[ X_{\text{z}}(\Omega) = X_{\text{z}} \cdot Z_{B3} \cdot 3 \]

Assumes 4
- If impedances are given in \( \Omega \)
  - Divide by appropriate \( Z_b \)

- If impedance is in per unit
  - Renormalize to system base
    - If the equipment base is different
      - Transformers, Electric Machines
      - Per unit or equipment rating base
\[ 3E \]

138 kV, 13.8 kV
200 MVA
\[ x = 0.1 \text{ pu} \]
\[ x/R = 10 \]

System

\[ V_{B1} = 132 \text{kV} \]
\[ V_{B2} = 132 \text{kV} \left( \frac{13.8}{138} \right) = 13.2 \text{kV} \]
\[ S_B = 100 \text{ MVA} \]

Change of base calculation

Option 1: Convert pu. value to ohms on either HV or LV winding.

Option 2: Convert to per unit on sys base on same side of xformer.
(2) Change to base equation

\[ X_{pu\ new} = X_{pu\ old} \left( \frac{V_{B\ old}}{V_{B\ new}} \right)^2 \left( \frac{5B_{new}}{5B_{old}} \right) \]

Loads - calculate equivalent impedance using \( \overline{5_{30}} \) and rated voltage
Per unit equivalent circuit