ECE 420: Homework #3

Due Date: Session 29 (April 7)

Solve the following problems

1. A round rotor synchronous generator (X_s = 1.0, r = 0) is synchronized to a bus whose voltage is 1.0 @0 deg. At synchronization i_f = 1000A (actual). The mechanical power is increased until P_G = 0.8pu. Now i_f is adjusted from 1000A to 1600A (with P_G and V_a held constant).

   (a) Plot a curve of |I_a| versus i_f
   (b) Plot a curve of Q_G versus i_f
   (c) When |I_a| is minimum, what is I_a (magnitude and angle)? What is the power factor?
   (d) Find X_s in ohms

2. A synchronous generator is used to supply a load drawing S=1.0 p.u. at a power factor of 0.85 lagging connected to the generator terminals. Assuming the terminal voltage is 1.05 p.u.

   (a) Determine E_o if the generator is a round rotor machine with synchronous reactance X_s = 1.1 p.u. and stator resistance R_a = 0.025pu. Sketch a per unit phasor diagram.
   (b) Repeat parts (a) if the machine is supplying a unity power factor load
   (c) Repeat parts (a) if the machine is supplying a load with a power factor of 0.85 leading and S = 1.0p.u.
   (d) Repeat parts (a) if the machine is operating as a synchronous condenser supplying S = 0 + j1.0pu
- \( L_{ab} \) is mutual inductance between phases A and B
- \( L_{ac} \) is mutual inductance between phases A and C
- \( L_{af} \) is mutual inductance between phases A the field winding F

We will find later that each of these inductances each have a constant part and a part that varies with time as the rotor turns.

- As a first approximation, we can break the \( L_{aa} \) into:
  \[
  L_{aa} = L_{aa0} + L_{al}
  \]
  where:
  \[
  L_{aa0} = \frac{N_s^2}{2 \cdot R_{lag}} \\
  L_{al} = \text{Leakage}
  \]

  \[
  L_{ab} = L_{aa0} \cdot \cos(120\text{deg})
  \]
  Note that only the self term has leakage (leakage is not part of the mutual inductance.

  \[
  L_{ac} = L_{aa0} \cdot \cos(-120\text{deg})
  \]

  \[
  L_{af} = L_f \cdot \cos(\theta_0 + \omega \cdot t)
  \]
  note that this varies with time.

  \[
  L_f = \frac{N_f \cdot N_s}{2 R_{lag}}
  \]

Then we can rewrite the flux linkage equation as:

\[
\lambda_a = L_{aa0} \left[ i_a(t) - \left( \frac{i_b(t)}{2} - \frac{i_c(t)}{2} \right) \right] + L_{al} \cdot i_a(t) + L_f \cdot i_f \cdot \cos(\theta_0 + \omega \cdot t)
\]

- Note impact of \( \cos(120) \) and \( \cos(-120) \)

Assume balanced three phase circuit:

\[
i_a + i_b + i_c = 0 \quad \text{or:} \quad i_a = -(i_b + i_c)
\]

So we can rewrite the expression as:

\[
\lambda_a = \frac{3}{2} \cdot L_{aa0} \cdot i_a(t) + L_{al} \cdot i_a(t) + L_f \cdot i_f \cdot \cos(\theta_0 + \omega \cdot t)
\]
Then the voltage equation becomes:

\[ v_a(t) = r_a \cdot i_a(t) + \frac{d}{dt} \lambda_a = r_a \cdot i_a(t) + \frac{3}{2}L_{aa0} \frac{d}{dt} i_a(t) + L_{al} \frac{d}{dt} i_a(t) - \omega \cdot L_F \cdot i_F \cdot \sin(\theta_0 + \omega \cdot t) \]

Define:

Voltage due to \( B_r \)

\[ e_a(t) = -\omega \cdot L_F \cdot i_F \cdot \sin(\theta_0 + \omega \cdot t) = \omega \cdot L_F \cdot i_F \cdot \cos(\frac{\pi}{2} + \theta_0 + \omega \cdot t) \]

Define:

\[ \delta = \frac{\pi}{2} + \theta_0 \]

Phasor form:

\[ \bar{E}_a = |E_a| e^{j \cdot \delta} \]

• Voltage due to \( B_s \) (armature reaction)

\[ \frac{3}{2}L_{aa0} \frac{d}{dt} i_a(t) \]

Now define the **direct axis synchronous reactance**:

\[ X_s = X_d = 2\cdot\pi \cdot 60 \text{ Hz} \left( \frac{3}{2}L_{aa0} + L_{al} \right) \]

Dominated by \( L_{aa0} \) since leakage is small

So the voltage equation becomes:

\[ v_a(t) = r_a \cdot i_a(t) + L_s \frac{d}{dt} i_a(t) + e_a(t) \]

Or in phasor form:

\[ \bar{V}_a = \bar{r}_a \bar{i}_a + j \cdot X_s \bar{i}_a + \bar{E}_a \]

Think back to dc machine, this implies current entering machine (motor operation)