ECE 420: Homework #3

Due Date: Session 29 (April 1)

Solve the following problems

1. A round rotor synchronous generator \((X_s = 1.0, r=0)\) is synchronized to a bus whose voltage is 1.0 @0 deg. At synchronization \(i_f = 1000\)A (actual). The mechanical power is increased until \(P_G = 0.8\)pu. Now \(i_f\) is adjusted from 1000A to 1600A (with \(P_G\) and \(V_a\) held constant).

(a) Plot a curve of \(|I_a|\) versus \(i_f\)
(b) Plot a curve of \(Q_G\) versus \(i_f\)
(c) When \(|I_a|\) is minimum, what is \(I_a\) (magnitude and angle)? What is the power factor?
(d) Find \(X_s\) in ohms

2. A synchronous generator is used to supply a load drawing \(S=1.0\) p.u. at a power factor of 0.85 lagging connected to the generator terminals. Assuming the terminal voltage is 1.05 p.u.

(a) Determine \(E_a\) if the generator is a round rotor machine with synchronous reactance \(X_s=1.1\) p.u. and stator resistance \(R_a = 0.025\)pu. Sketch a per unit phasor diagram.

(b) Repeat parts (a) if the machine is supplying a unity power factor load
(c) Repeat parts (a) if the machine is supplying a load with a power factor of 0.85 leading and \(S = 1.0\)p.u.
(d) Repeat parts (a) if the machine is operating as a synchronous condenser supplying \(S = 0 + j1.0\)pu
Then the voltage equation becomes:

\[ v_a(t) = r_a \cdot i_a(t) + \frac{d}{dt} \lambda_a = r_a \cdot i_a(t) + \frac{3}{2} \cdot L_{aa0} \cdot \frac{d}{dt} i_a(t) + L_{al} \cdot \frac{d}{dt} i_a(t) - \omega \cdot L_F \cdot i_F \cdot \sin(\theta_0 + \omega \cdot t) \]

Define:

- Voltage due to \( B_r \)

\[ e_a(t) = -\omega \cdot L_F \cdot i_F \cdot \sin(\theta_0 + \omega \cdot t) = \omega \cdot L_F \cdot i_F \cdot \cos\left(\frac{\pi}{2} + \theta_0 + \omega \cdot t\right) \]

Define:

\[ \delta = \frac{\pi}{2} + \theta_0 \]

Phasor form:

\[ \vec{E}_a = |E_a| \cdot e^{j \cdot \delta} \]

- Voltage due to \( B_s \) (armature reaction)

\[ \frac{3}{2} \cdot L_{aa0} \cdot \frac{d}{dt} i_a(t) \]

Now define the **direct axis synchronous reactance**:

\[ X_s = X_d = 2 \cdot \pi \cdot 60 \text{Hz} \left(\frac{3}{2} \cdot L_{aa0} + L_{al}\right) \]

Dominated by \( L_{aa0} \) since leakage is small.

So the voltage equation becomes:

\[ v_a(t) = r_a \cdot i_a(t) + L_s \cdot \frac{d}{dt} i_a(t) + e_a(t) \]

Or in phasor form:

\[ \vec{V}_a = r_a \cdot \vec{i}_a + j \cdot X_s \cdot \vec{i}_a + \vec{E}_a \]

Think back to dc machine, this implies current entering machine (motor operation)
Generator equation:

\[ V_a = E_a - r_a I_a - j X_s I_a \]

Per Phase Equivalent Circuit (assumes Y connected):

- For a large machine it is generally possible to neglect \( R_a \)
- Normally the X/R ratio is over 20
- For the smaller machines in our lab, the ratio will be smaller and the resistance will be significant

Large machines grounded through resistor to limit fault current
Control of synchronous machine

- Inputs: If
  \[ P_{\text{mech}} \rightarrow w_m \]

In equivalent circuit
- If \( \text{controlling LEA} \)
  \[ \text{indirect impact on } \delta \]

\[ P_{\text{mech}} \rightarrow \text{effectively controls } \delta \]
- If maintain synch speed
  \[ w_m \text{ largely fixed} \]
IF $P_{elec} > P_{mech}$ in "steady state"
Pull kinetic energy out of rotor
(slow down)
field voltage supply

1. DC generator

so output to controllers
controls field current of
DC machine

2. Solid state excitors

\[ f \]

Ac voltage controller rectifier
The diagram shows a circuit with a thyristor labeled as 'Thyristor'. The equation $V_{dc} = k |V_{ac}| \cos (\alpha)$ is written, where $\alpha$ is the firing delay angle. $\alpha = 0$ is where a diode would turn on.
For the moment let $R_A = 0$

$$P_{terminal} = R_e \left( VA \cdot \overline{I_A} \right)\text{pu}$$

$$= TRe \left( VA \cdot \left( \frac{E_A - VA}{jX_s} \right)^* \right)$$

$$= Re \left[ \frac{VA}{X_s} \text{Re} \left( \Theta_{VA} - \Theta_{EA} + 90 \right) - \frac{VA^2}{X_s} \right] \text{pu}$$
\[ \frac{1 \text{Vall/Ea}}{X_5} \cos \cos (\Theta_{Va} - \Theta_{Ea} + 90^\circ) = \sin \]

\[ p = - \frac{1 \text{Vall/Ea}}{X_5} \sin (\Theta_{Va} - \Theta_{Ea}) \]

\[ p = \frac{1 \text{Vall/Ea}}{X_5} \sin(\delta) \]
Power out of machine
\[ \delta > 0 \] - generator

Power into machine
\[ \delta < 0 \] - motor