Chapter 4

Synchronous Generators
Basic Topology

• In *stator*, a three-phase winding similar to the one described in chapter 4. Since the main voltage is induced in this winding, it is also called *armature winding*.

• In *rotor*, the magnetic field is generated either by a permanent magnet or by applying dc current to rotor winding. Since rotor is producing the main field, it is also called *field winding*. Two rotor designs are common:
- Salient-pole rotor with “protruding” poles

- Round or Cylindrical rotor with a uniform air gap
Figure 4-3
Brushless exciter circuit.
Figure 4-5
A brushless excitation scheme that includes a pilot exciter. The permanent magnets of the pilot generator produces a field current of the exciter which in turns produces the field current of the main machine.
The Speed of Rotation of a Synchronous Generator

\[ f_e = \frac{n_m P}{120} \]

Where

\[ f_e = \text{electrical frequency, in Hz} \]

\[ n_m = \text{mechanical speed of magnetic field, in rpm} \]

\[ = \text{rotor speed, in rpm} \]

\[ P = \text{number of poles} \]
The Internal Generated Voltage of a Synchronous Generator

• It was shown previously, the magnitude of the voltage induced in a given stator phase was found to be

\[ E_A = \sqrt{2\pi} N_c \phi f = \frac{N_c \phi}{\sqrt{2}} \omega \]

• The induced voltage is proportional to the rotor flux for a given rotor angular frequency in electrical Radians per second.

• Since the rotor flux depends on the field current \( I_F \), the induced voltage \( E_A \) is related to the field current as shown below. This is generator magnetization curve or the open-circuit characteristics of the machine.
Figure 4-7
(a) Plot of flux versus field current for a synchronous generator. (b) The magnetization curve for the synchronous generator.
The Equivalent Circuit of a Synchronous Generator

• When generator is not loaded, the internal generated voltage $E_A$ is the same as the voltage appearing at the terminals of the generator, $V_\phi$.

• When generator is loaded, a balanced 3-phase current will flow which results in the stator rotating magnetic field $B_S$. The net air gap flux density is the sum of the rotor and stator magnetic fields:

$$B_{net} = B_R + B_S$$
Figure 4-8
• The voltage induced in the armature would be the sum of the voltages induced by rotor field \((E_A)\) and the voltage induced by the stator field \((E_{AR}, \text{ or armature-reaction voltage})\).

\[
E_{net} = E_A + E_{AR}
\]

Two other voltage drops must be considered:

- Self (or leakage) inductance of the armature coils.
- Resistance of the armature coils

• The armature-reaction voltage may be represented by an inductive voltage drop across an armature–reaction reactance \(X_{AR}\), as shown here.
The two reactances may be combined into a single reactance called the synchronous reactance of the machine:

\[ X_S = X_{AR} + X_I \]

Figure 4-12
The per phase equivalent circuit of a synchronous generator.
The Phasor Diagram of a Synchronous Generator

• The Kirchhoff’s voltage law equation for the armature circuit is

\[ E_A = V_\phi + I_A (R_A + jX_S) \]

• The phasor diagrams for unity, lagging, and leading power factors load are shown here:
Power and Torque in Synchronous Generators

\[ P_{\text{in}} = \tau_{\text{app}} \omega_m \]

\[ \tau_{\text{ind}} \omega_m \]

\[ P_{\text{conv}} \]

\[ P_{\text{out}} = \sqrt{3} V_L I_L \cos \theta \]

\[ I^2R \text{ losses (copper losses)} \]

\[ \text{Stray losses} \]

\[ \text{Friction and windage losses} \]

\[ \text{Core losses} \]

Figure 4-15
The power-flow diagram of a synchronous generator
• The input mechanical power is given by

\[ P_{in} = \tau_{app} \omega_m \]

• The power converted from mechanical to electrical power is given by

\[ P_{conv} = \tau_{ind} \omega_m = E_A I_A \cos(\gamma) \]

• The real and reactive electrical output power is given by

\[ P_{OUT} = 3V \phi_A \cos(\theta) \]
\[ Q_{OUT} = 3V \phi_A \sin(\theta) \]
If the armature resistance is ignored (Since \( R_A \ll X_S \)),

\[
I_A \cos(\theta) = \frac{E_A \sin(\delta)}{X_S}
\]

\[
P_{CONV} = P_{OUT} = \frac{3V \phi E_A \sin(\delta)}{X_S}
\]

Figure 4-16

Simplified phasor diagram with armature resistance ignored
• Induced torque of the generator is given by

\[ \tau_{ind} = \frac{3V\phi E_A \sin(\delta)}{\omega_m X_s} \]

• Note that this equation offers an alternative form for the induced torque presented before by

\[ \tau_{ind} = KB_{net} B_R \sin(\delta) \]

• Since \( B_{net} \) produces \( V_\phi \) (assuming negligible \( R_A \) and leakage reactance) and \( B_R \) produces \( E_A \), power angle \( \delta \) between \( E_A \) and \( V_\phi \) is the same as the angle \( \delta \) between \( B_R \) and \( B_{net} \).
Measuring Synchronous Generator Parameters

• Open-circuit and short-circuit tests to obtain magnetization characteristics and synchronous reactance of the generator.

  o Open-circuit test: With loads disconnected, generator is driven at rated speed. The terminal voltage is measured as field current varied.

  o Short-circuit test: Armature terminals shorted, generator is driven at rated speed and the armature current is measured as field current varied.

• DC voltage test to obtain the armature resistance.
Figure 4-17
(a) The open-circuit characteristics (OCC) of a synchronous generator. (b) The short-circuit characteristics (SCC) of a synchronous generator.
Steps to obtain unsaturated synchronous reactance $X_{su}$ at a given field current:

1. Get $E_A$ from air-gap line on OCC
2. Get the armature at the same $I_f$ from SCC

$$X_{su} = \frac{E_A|_{OCC,Agline}}{I_A|_{SCC}}$$

Figure 4-19
A sketch of the approximate synchronous reactance of a synchronous generator as a function of the field current.
The Effect of Load Changes on a Synchronous Generator Operating Alone

- At constant field current and rotor speed

Figure 4-22
The effect of an increase in generator load upon its terminal voltage. At a fixed power factor (a) Lagging; (b) unity; (c) leading.

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Parallel Operation of Synchronous Generators

Requirements:
1. Must have the same \textit{voltage magnitude}.
2. The \textit{phase angles} of the two a phases must be the same.
3. The generators must have the same \textit{phase sequences}.
4. The \textit{frequency} of the oncoming generator must be slightly higher than the frequency of the running generator.

Figure 4-27
(a) The two possible phase sequences of a three phase system
(b) The three-light-bulb method for checking phase sequence.

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Frequency-Power Characteristics of a Synchronous Generator

\[ P = S_P (f_{nl} - f_{sys}) \]

Figure 4-29
(a) The speed-power curve for a typical prime mover. (b) The resulting frequency-power curve for the generator.
Operation of Synchronous Generators in Parallel with Large Power Systems

• Since infinite bus has a constant voltage and frequency, its $f-P$ and $V-Q$ characteristics are horizontal lines.

Figure 4-33
(a) A synchronous generator operating in parallel with an infinite bus. (b) The $f-P$ diagram (or house diagram) for a synchronous generator in parallel with an infinite bus.

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Figure 4-36
The effect of increasing the governor’s set point on at constant excitation (a) the house diagram; (b) the phasor diagram.
Figure 4-37
The effect of increasing the generator’s field current at constant power on the phasor diagram of the machine
Summary

When a generator is operating in parallel with an infinite bus:

1. The frequency and terminal voltage of the generator are controlled by the system to which it is connected.
2. The governor set points of the generator control the real power \((P)\) supplied by the generator to the system.
3. The field current in the generator controls the reactive power \((Q)\) supplied by the generator to the system.
Operation of Synchronous Generators in Parallel with Other Generators of the Same Size

Figure 4-38
(a) A generator connected in parallel with another machine of the same size. (b) The corresponding house diagrams at the moment generator 2 is paralleled with the system. (c) The effect of increasing generator 2’s governor set point on the operation of the system. (d) The effect of increasing generator 2’s field current on the operation of the system.
Operation of Synchronous Generators in Parallel with Other Generators of the Same Size

Figure 4-40
(a) Shifting power sharing without affecting system frequency. (b) Shifting system frequency without affecting power sharing. (c) Shifting reactive power sharing without affecting terminal voltage. (d) Shifting terminal voltage without affecting reactive power sharing.

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Summary

In case of two generators operating together:
1. The system is constrained in that the total power supplied by the two generators together must equal the total amount consumed by the load. The $f_{sys}$ is not constrained to be constant.
2. To adjust the real power ($P$) sharing between generators without changing $f_{sys}$, simultaneously increase the governor set point on one generator while decreasing the governor set point on the other. The machine whose governor set point was increased will assume (undertake) more of the load.
3. To adjust $f_{sys}$ without changing the real power sharing, simultaneously increase or decrease both generators’ governor set points.
Synchronous Generator Transients

• Static stability limit is the maximum power generator can supply under *gradual* load change.

\[ P_{\text{max}} = \frac{3V \phi E_A}{X_S} \]

• Transient stability limit is the maximum power generator can supply under *sudden* load change. The machine’s reactance during these changes is different than the one during steady-state condition.
Short-Circuit Transients in Synchronous Generators

\[ i(t) = \sqrt{2} \left( (I'' - I') e^{-t/T''} + (I' - I_{ss}) e^{-t/T'} + I_{ss} \right) \cos(\omega t + \theta_o) \]

Figure 4-45
The Symmetric ac component of the fault current.
• Steady-state, transient, and subtransient short-circuit currents when a sudden short is applied across the terminals of an *unloaded* generator.

\[
I_{ss} = \frac{E_A}{X_s}
\]

\[
I' = \frac{E_A}{X'}
\]

\[
I'' = \frac{E_A}{X''}
\]
Figure 4-46
A semilogarithmic plot of the magnitude of the ac component of the fault current as a function of time. The subtransient and transient time constants of the generator can be determined from such plots.
Synchronous Generator Ratings

- Armature heating sets the limit on the armature current, independent of the power factor

\[ P_{SCL} = 3I_A^2 R_A \]

- For a given rated voltage, the maximum acceptable \( I_A \) determines the rated KVA of the generator

\[ S_{\text{rated}} = 3V_{\phi,\text{rated}} I_{A,\text{max}} = \sqrt{3}V_{L,\text{rated}} I_{L,\text{max}} \]
Synchronous Generator Ratings

• The rotor heating sets the limit on the machine’s field current and hence sets the maximum allowable $E_A$ and rated power factor

Figure 4-47
The effect of the rotor field current limit on setting the rated power factor of the generator
Synchronous Generator Capability Curve

Figure 4-48
Derivation of a synchronous generator capability curve. (a) The generator phasor diagram; (b) the corresponding power limits.

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Figure 4-50
A capability diagram

$$Q = -\frac{3V_\phi^2}{X_s}$$
Salient-Pole Generators

The power output of a synchronous generator with a cylindrical rotor was given as:

\[ P_{\text{CONV}} = P_{\text{OUT}} = \frac{3V\phi E_A \sin(\delta)}{X_S} \]

What is the power output of a salient-pole generator?
Salient-Pole Generators

\[ P = 3 \, V_\phi \, I_d \cos (90^\circ - \delta) + 3 \, V_\phi \, I_q \cos \delta \]

**FIGURE C-6**
Determining the power output of a salient-pole synchronous generator. Both \( I_d \) and \( I_q \) contribute to the output power, as shown.
Salient-Pole Generators

\[ P = 3V_\phi I_d \sin \delta + 3V_\phi I_q \cos \delta \]

\[ I_d = \frac{E_A - V_\phi \cos \delta}{X_d} \]

\[ I_q = \frac{V_\phi \sin \delta}{X_q} \]
Salient-Pole Generators

\[ P = 3V_\phi I_d \sin \delta + 3V_\phi I_q \cos \delta \]

\[
P = 3V_\phi \left( \frac{E_A - V_\phi \cos \delta}{X_d} \right) \sin \delta + 3V_\phi \left( \frac{V_\phi \sin \delta}{X_q} \right) \cos \delta
\]

\[
= \frac{3V_\phi E_A}{X_d} \sin \delta + 3V_\phi^2 \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin \delta \cos \delta
\]

Since \( \sin \delta \cos \delta = \frac{1}{2} \sin 2\delta \), this expression reduces to

\[
P = \frac{3V_\phi E_A}{X_d} \sin \delta + \frac{3V_\phi^2}{2} \left( \frac{X_d - X_q}{X_d X_q} \right) \sin 2\delta
\]
Salient-Pole Generators

\[ P = \frac{3V_{\phi}E_A}{X_d} \sin \delta + \frac{3V_{\phi}^2}{2} \left( \frac{X_d - X_q}{X_dX_q} \right) \sin 2\delta \]  \hspace{1cm} (C-12)

Since the induced torque in the generator is given by \( \tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} \), the induced torque in the motor can be expressed as

\[ \tau_{\text{ind}} = \frac{3V_{\phi}E_A}{\omega_m X_d} \sin \delta + \frac{3V_{\phi}^2}{2\omega_m} \left( \frac{X_d - X_q}{X_dX_q} \right) \sin 2\delta \]  \hspace{1cm} (C-13)