If a load is unbalanced, its neutral, m, will not be at the same potential as the source neutral, n. Derive the relationship between the neutral shift $V_{mn}$ and the zero sequence voltage $V_{am0}$ for the system shown below. Similar to problem 2.11. Hint, consider the line to ground voltages, line to neutral voltages and neutral to ground voltages.

\[ V_{m_n} = V_{m_{ground}} - V_{n_{ground}} \]
\[ V_{an} = V_{a_{ground}} - V_{n_{ground}} \]
\[ V_{am} = V_{a_{ground}} - V_{m_{ground}} \]

so, substituting
\[ V_{m_n} = V_{m_{ground}} - V_{n_{ground}} = (V_{a_{ground}} - V_{am}) - (V_{a_{ground}} - V_{am}) \]

Then rearranging terms:
\[ V_{m_n} = V_{an} - V_{am} \]

similarly
\[ V_{m_n} = V_{bn} - V_{bm} \]
\[ V_{m_n} = V_{cn} - V_{cm} \]
• Adding these three expressions, results in:
  \[ 3V_{m_n} = (V_{an} + V_{bn} + V_{cn}) - (V_{am} + V_{bm} + V_{cm}) \]

• Applying the Symmetrical Components transformation:
  \[ 3V_{an0} = (V_{an} + V_{bn} + V_{cn}) = 0 \]
  Since the source is still balanced

  Similarly
  \[ 3V_{am0} = (V_{am} + V_{bm} + V_{cm}) \]
  This does not sum to 0, since the load is unbalanced

  Therefore
  \[ 3V_{m_n} = 3V_{an0} - 3V_{am0} = -3V_{am0} \]
  \[ V_{m_n} = -V_{am0} \]

As a check, the circuit was simulated with ATPDraw, which was also used to determine the symmetrical instantaneous quantities.

• First we see \( V_{ag}, V_{bg}, V_{cg}, V_{ng} \) and \( V_{mg} \) (brown line)
• Note the unbalance in the line-to-ground voltages.

![Graph showing line-to-ground voltages with ATPDraw simulation](file:lect6example.pl4; x-var t) v:VA v:VB v:VC v:VN v:VM

![Graph](file:lect6example.pl4; x-var t) v:VA v:VB v:VC v:VN v:VM
- Next plot instantaneous values of $V_{mn}$ and $V_0$ (note that $V_0$ is simply $1/3*(V_{am} + V_{bm} + V_{cm})$). No phase degree phase difference.

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
 v (A) & -2000 & -1500 & -1000 & -500 & 0 & 500 & 1000 & 1500 & 2000 & \\
\end{array}
\]

- The capacitors were added to provide a ground reference. The capacitive reactances are quite large and don’t impact the results otherwise.
- Control modelling language TACS used to calculate $V_0$. 

ATPDraw Schematic

Measure voltage across large resistor