For the simple 2 bus system shown in the figure look at steady-state reactive compensation

|V1| = 132 kV (line to line) at an angle of 0 degrees

|V2| = 132 kV (line to line) at an angle of -20.

Define Constants

Sending and Receiving End Voltages

\[ V_{1L} := 132kV \]
\[ V_{1p} := \frac{V_{1L}}{\sqrt{3}} \]
\[ V_{1p} = 76.21\cdot kV \]

\[ V_{2L} := 132kV \]
\[ V_{2p} := \frac{V_{2L}}{\sqrt{3}} \]
\[ V_{2p} = 76.21\cdot kV \]

Sending and Receiving End Angles

\[ \theta_1 := 0\deg \]
\[ \theta_2 := -20.14\deg \]

Line Reactance

\[ X_{line} := 10\text{ohm} \]

Define Unit for MVA

\[ \text{MVA} := \text{MW} \]
(a) Compute $P_{12}$

$$P_{12} := \left(\frac{3 \cdot V_{1p} \cdot V_{2p}}{X_{\text{line}}}\right) \cdot \sin(\theta_1 - \theta_2)$$

$$P_{12} = 599.93\text{ MW} \quad \text{For all intensive purposes: 600 MW}$$

(b) Next increase $P_{12}$ to 700 MW. Consider the following options to do so. Give a written evaluation of each form of compensation

A. Add series capacitance. What size capacitor is needed? What is the percent compensation? If the max compensation is 60%, what is $P_{12\ max}$?

For all intensive purposes: 600 MW

$$X_{\text{total}} := \frac{3 \cdot V_{1p} \cdot V_{2p} \cdot \sin(\theta_1 - \theta_2)}{700\text{ MW}} \quad X_{\text{total}} = 8.57\text{ ohm}$$

$$X_{\text{cap}} := X_{\text{line}} - X_{\text{total}} \quad X_{\text{cap}} = 1.43\text{ ohm}$$

$$\text{Percent Compensation} := \frac{X_{\text{cap}}}{X_{\text{line}}} \quad \text{Percent Compensation} = 14.3\%$$

$$\text{Cap} := \frac{1}{2 \cdot \pi \cdot 60\text{Hz} \cdot X_{\text{cap}}} \quad \text{Cap} = 1855.6\ \mu\text{F}$$
\[
X_{c\_max} := 0.6 \cdot X_{\text{line}} \\

P_{12\_max} := \frac{3 \cdot V_{1p} \cdot V_{2p} \cdot \sin(\theta_1 - \theta_2)}{X_{\text{line}} - X_{c\_max}} \quad P_{12\_max} = 1499.84 \text{ MW}
\]

**Advantages**
1) Effective way to increase power transfer
2) Can vary compensation by switching multiple banks in series/parallel combinations
3) Insensitive to location
4) Relatively inexpensive (compared to other options)
5) Doesn't result in overvoltages at either bus when line lightly loaded

**Disadvantages**
1) Potential series resonance problems
2) Possible subsynchronous resonance (SSR)
3) Stepwise variation in compensation only
4) Slow response if need to change \(X_c\) (can't damp slow oscillations)
5) Overvoltage protection needed for capacitor in event of fault on line
6) Impact on distance protection schemes

**B.** Instead of series compensation add phase angle control. What phase, \(\alpha\) does the compensator need to produce? Lets say that \(\alpha_{\text{max}} = 30^\circ\), what is \(P_{12\_max}\)?

![Diagram](attachment:image.png)

- \(|V1| = 132 \text{ kV (line to line)}\) at an angle of 0 degrees
- \(|V2| = 132 \text{ kV (line to line)}\) at an angle of -20.1 degrees
TotalAngle := \text{asin} \left( \frac{700 \text{MW} \cdot X_{\text{line}}}{3 \cdot V_{1p} \cdot V_{2p}} \right) \quad \text{TotalAngle} = 23.69 \text{ deg}

\alpha := \text{TotalAngle} - (\theta_1 - \theta_2) \quad \alpha = 3.55 \text{ deg}

\alpha_{\text{max}} := 30 \text{ deg}

\begin{equation}
\begin{split}
P_{12_{\text{maxb}}} := & \frac{3 \cdot V_{1p} \cdot V_{2p} \cdot \sin \left( (\theta_1 - \theta_2) + \alpha_{\text{max}} \right)}{X_{\text{line}}} \\
& \quad \text{P}_{12_{\text{maxb}}} = 1337.49 \text{ MW}
\end{split}
\end{equation}

\textbf{Advantages}
1) Effective control of power flow in a given line
2) Can locate at substation at the end of a line
3) Unlikely to create resonance problems

\textbf{Disadvantages}
1) Potential stability problems at large angles
2) Somewhat sensitive to location
3) Stepwise variation in compensation only
4) Slow response if need to change \alpha (can't damp slow oscillations)
C. We next look at adding midpoint compensation. Let's just call it an ideal compensator right now. What midpoint voltage is needed? How much reactive power does the compensator provide? Do you consider this a feasible solution?

\[ X_{\text{half}} := \frac{X_{\text{line}}}{2} \quad X_{\text{half}} = 5\cdot\text{ohm} \]

Lossless line, so the following relationships hold:

\[ P_{12\text{req}} := 700\text{MW} \]
\[ P_{1m} := P_{12\text{req}} \quad P_{m2} := P_{12\text{req}} \]

\[ \theta_m := \frac{\theta_2 - \theta_1}{2} \]

\[ V_{\text{mp}} := \frac{P_{1m} \cdot X_{\text{half}}}{3 \cdot V_1 \cdot \sin(\theta_1 - \theta_m)} \]

\[ V_{\text{mp}} = 87.55\text{kV} \]
\[
V_{mL} := \sqrt{3} \cdot V_{mp} \quad V_{mL} = 151.64 \text{-kV}
\]

\[
\text{OverVolt} := \frac{V_{mL}}{V_{1L}} \quad \text{OverVolt} = 114.88 \% 
\]

\[
Q_{m1} := \left( \frac{V_{mL}}{X_{\text{half}}} \right)^2 - \left( \frac{V_{mL} \cdot V_{1L} \cdot \cos(\theta_m - \theta_1)}{X_{\text{half}}} \right) \quad Q_{m1} = 657.45 \text{ MVAr}
\]

\[
Q_{m2} := \left( \frac{V_{mL}}{X_{\text{half}}} \right)^2 - \left( \frac{V_{mL} \cdot V_{2L} \cdot \cos(\theta_m - \theta_2)}{X_{\text{half}}} \right) \quad Q_{m2} = 657.45 \text{ MVAr}
\]

\[
Q_{\text{compensator}} := Q_{m1} + Q_{m2}
\]

\[
Q_{\text{compensator}} = 1314.91 \text{ MVAr}
\]

This is almost double P12, so this is not a reasonable solution in this case.

**Advantages**
1) Local reactive support
2) Will not create resonance problems unless use fixed capacitors
3) Continuously variable is use synchronous condensor

**Disadvantages**
1) Sensitive to location
2) Relatively slow response if rotating machine
3) Not as effective as series cacapitor or phase shifting transformer
4) High cost
Example: For the system below we want to increase the power transfer from Bus 1 to Bus 2 to 650 MW by adding a STATCOM at the midpoint. The STATCOM is connected through a 10:1 transformer with a leakage reactance of 0.02 ohm on low voltage side, determine the MVA, voltage, and current supplied by the STATCOM. Simulate in a powerflow program and in ATP (using an idea source for the STATCOM for now).

\[ |V_1| = 345kV \] at 0°
\[ |V_m| = ?? \] at \((\theta_1 - \theta_2)/2\)
\[ |V_2| = 345kV \] at -17.605°

- Convert to per unit for Powerworld simulation:
  \[ V_{LL} := 345kV \quad S_B := 100MVA \quad Z := \frac{V_{LL}^2}{S_B} \quad Z_B = 1190.25 \Omega \]

- Sending and Receiving End Voltages
  \[ V_{1L} := 345kV \quad V_{1p} := \frac{V_{1L}}{\sqrt{3}} \quad V_{1p} = 199.19\text{-kV} \]
  \[ V_{2L} := 345kV \quad V_{2p} := \frac{V_{2L}}{\sqrt{3}} \quad V_{2p} = 199.19\text{-kV} \]
• Sending and Receiving End Angles
  \[ \theta_1 := 0\text{deg} \quad \theta_2 := -17.605\text{deg} \]

• Line Reactance
  \[ X_{\text{line}} := 60\text{ohm} \]

• Per unit line reactance
  \[ Z_{\text{line}pu} := \frac{j \cdot X_{\text{line}}}{Z_B} \quad Z_{\text{line}pu} = 0.05i\text{pu} \]

• Power transfer without the shunt compensator
  \[ \frac{3V_{1p} \cdot V_{2p} \cdot \sin(\theta_1 - \theta_2)}{X_{\text{line}}} = 599.99\text{MW} \]

• Midpoint compensation set up
  \[ X_{\text{half}} := \frac{X_{\text{line}}}{2} \quad X_{\text{half}} = 30\text{ohm} \]
  \[ Z_{\text{half}pu} := \frac{j \cdot X_{\text{half}}}{Z_B} \quad Z_{\text{half}pu} = 0.03i\text{pu} \]
• Lossless line, so the following relationships holds:

\[ P_{12\text{req}} := 650\text{MW} \]

\[ P_{1m} := P_{12\text{req}} \quad P_{m2} := P_{12\text{req}} \]

\[ \theta_m := \frac{\theta_2 - \theta_1}{2} \quad \theta_m = -8.8\cdot\text{deg} \]

• Assuming that the angles at the ends of the line to not change with change in P12

\[ V_{mp} := \frac{P_{1m}\cdot X_{\text{half}}}{3\cdot V_{1p}\cdot \sin(\theta_1 - \theta_m)} \]

\[ V_{mp} = 213.25\cdot\text{kV} \quad V_{mL} := \sqrt{3}\cdot V_{mp} \quad V_{mL} = 369.35\cdot\text{kV} \]

\[ \text{OverVolt} := \frac{V_{mL}}{V_{1L}} \quad \text{OverVolt} = 107.06\% \]

\[ Q_{m1} := \frac{V_{mL}^2}{X_{\text{half}}} - \frac{V_{mL}\cdot V_{1L}\cdot \cos(\theta_m - \theta_1)}{X_{\text{half}}} \]

\[ Q_{m1} = 349.86\cdot\text{MVAR} \]

\[ Q_{m2} := \frac{V_{mL}^2}{X_{\text{half}}} - \frac{V_{mL}\cdot V_{2L}\cdot \cos(\theta_m - \theta_2)}{X_{\text{half}}} \]

\[ Q_{m2} = 349.86\cdot\text{MVAR} \]

\[ Q_{\text{compensator}} := Q_{m1} + Q_{m2} \quad \text{This is the net capacitive contribution.} \]

\[ Q_{\text{compensator}} = 699.72\cdot\text{MVAR} \]
• Now determine what the STATCOM ac terminal voltage will be and the current at the terminals.

\[
V_{\text{low}} := V_{\text{mL}} \cdot \frac{1}{10} \quad V_{\text{low}} = 36.94 \cdot \text{kV}
\]

\[
X_{\text{xfmr_low}} := 0.02 \text{ohm} \quad L_{\text{xfmr_low}} := \frac{X_{\text{xfmr_low}}}{2 \cdot \pi \cdot 60 \text{Hz}} \quad L_{\text{xfmr_low}} = 0.05 \cdot \text{mH}
\]

\[
Z_{\text{BLV}} := \frac{(34.5 \text{kV})^2}{100 \text{MVA}} \quad X_{\text{pu}} := \frac{X_{\text{xfmr_low}}}{Z_{\text{BLV}}} \quad X_{\text{pu}} = 0.00168
\]

• In the steady-state, the STATCOM is equivalent to a voltage source behind an inductance (the transformer reactance). The fundamental component of the compensator voltage is in phase with the system voltage at the midpoint since no real power is transferred.

• We can take the equation for Q transferred across the transformer inductance (viewed from the line side, since the transformer absorbs Q too):

\[
Q_{\text{compensator}} = -\frac{V_{\text{low}}^2 - V_{\text{low}} \cdot V_{\text{comp}} \cdot \cos(0 \text{deg})}{X_{\text{xfmr_low}}} \quad \text{Negative sign since Q going to power system}
\]

\[
V_{\text{comp}} := \frac{Q_{\text{compensator}} \cdot X_{\text{xfmr_low}} + V_{\text{low}}^2}{V_{\text{low}} \cdot \cos(0 \text{deg})}
\]

\[
V_{\text{comp}} = 37.31 \cdot \text{kV} \quad \text{line to line} \quad \frac{V_{\text{comp}}}{\sqrt{3}} = 21.54 \cdot \text{kV}
\]

\[
\frac{V_{\text{comp}}}{34.5 \text{kV}} = 1.08157 \cdot \text{pu}
\]
Therefore $I_{\text{comp\_low}}$ can be found from:

$$I_{\text{comp\_low}} := \frac{Q_{\text{comp\_low}}}{\sqrt{3} \cdot V_{\text{low}}}$$

$$Q_{\text{statcom}} := \sqrt{3} \cdot V_{\text{comp\_low}} I_{\text{comp\_low}}$$

Alternate Approach:

$$Q_{HV\_xfmr} := Q_{\text{comp\_low}}$$

$$I_{HV\_side} := \left(\frac{Q_{HV\_xfmr}}{3 \cdot V_{mp}}\right) e^{j(\theta_m + 90\deg)}$$

$$I_{HV\_side} = 1093.76 \text{ A} \quad \arg(I_{HV\_side}) = 81.2\deg$$

$$I_{LV\_side} := I_{HV\_side} \cdot 10$$

$$|I_{LV\_side}| = 1.09 \times 10^4 \text{ A}$$

$$|I_{LV\_side}| - |I_{LV\_side}| = 0 \text{ A}$$

$$V_{t\_STATCOM\_LN} := \frac{V_{mp} e^{j\theta_m}}{10} - I_{LV\_side} e^{jX_{xfmr\_low}}$$

$$|V_{t\_STATCOM\_LN}| = 21.54 \text{ kV}$$

$$\arg(V_{t\_STATCOM\_LN}) = -8.8\deg$$

$$V_{comp} - \sqrt{3} |V_{t\_STATCOM\_LN}| = 0 \text{ V}$$

$$Q_{\text{statcom\_alt}} := \text{Im}\left(3 \cdot V_{t\_STATCOM\_LN} I_{LV\_side}\right)$$

$$Q_{\text{statcom\_alt}} = -706.9 \text{ MVAR}$$
**Powerworld Simulation Results:**

Basecase, no compensation, verify calculations.

![Diagram showing powerflow and voltage angles](attachment:diagram.png)

- Real power flow
- and BUS2 voltage angle are as expected

Case 1: With the step down transformer (preferred solution)

![Diagram showing powerflow and voltage angles with STATCOM](attachment:diagram_with_statcom.png)

Results match calculation with small error for:
- Midpoint voltage
- Real power transfer
- Midpoint reactive power injection at STATCOM
- Reactive power injection to midpoint bus
Case 2: Without the step down transformer:

Results match calculation with small error for:
- Midpoint voltage
- Real power transfer
- Reactive power injection to midpoint bus

**ATPDraw Simulation Results:**

- Power circuit:
  1. STATCOM represented just as an ideal voltage source
  2. $P_{12}$ calculated in TACS as described in class
34.5 kV System

345 kV @ -90

TACS Voltage Measurements

TACS Current Measurements

345 kV @ -107.6

STATCOM

VSTAT

STATCOM current on HV side of transformer

Power transfer:

STATCOM current on HV side of transformer

650MW
HV Voltage at midpoint (STATCOM bus)

\[ V_{HV} := 213.216 kV \cdot e^{-j \cdot 8.784 \text{deg}} \]

\[ I_{HV} := 1093.9 A \cdot e^{j \cdot 81.258 \text{deg}} \]
\[ I_{LV} := 10 \cdot I_{HV} \]
\[ |I_{LV}| = 10.94 \text{kA} \]

We calculated: \( I_{\text{comp\_low}} = 10.94 \text{kA} \)

LV Voltage at midpoint (STATCOM bus)

\[ V_{LV} := 21.54 kV \cdot e^{-j \cdot 8.784 \text{deg}} \]

\[ Q_{\text{STAT}} := 3 V_{LV} \cdot I_{LV} \]
\[ |Q_{\text{STAT}}| = 706.878 \text{MVAR} \]
\[ \arg(Q_{\text{STAT}}) = -90.04 \text{deg} \]

We calculated: \( Q_{\text{statcom}} = 706.9 \text{MVA} \)