An Overview of the Operation of HVDC Classic Transmission Systems

ECE 529
Spring 2021

Steady-State HVDC Converter Representation

- Steady state equivalent circuit

- Have fast, direct control over $\alpha$ (firing delay angle)
- $V_{dc} = V_{do} \times \cos \alpha$ (firing delay angle) where $V_{do} = const \times |V_{LL}|$
- Some control of $|V_{ac}|$ with tap changing transformer
- DC current indirectly controlled by changing $\alpha$
Basic Six-Pulse Converter

- Based on line commutated, current source converter
- Thyristors used as devices
- Converter with stiff current source on dc side
- Stiff voltage source on ac side (turns off thyristors)
- Basic 6-pulse bridge:

```
\begin{align*}
\text{Basic 6-pulse Bridge:} \\
\text{Thyristors} & \quad \text{Smoothing Reactor} \\
\text{AC Side:} & \quad \text{DC Side:} \\
\text{AC Voltage} & \quad \text{DC Current} \\
\text{Transformer Inductance} & \quad \text{L_s} \\
\end{align*}
```

- Initially assume: 1) Ideal ac sources, 2) ideal switches, 3) $X_c = 0$, and 4) $L_s \rightarrow \infty$ source
- AC side of converter has an ideal voltage source, dc side of converter has an ideal current source
- Apply Kirchhoff’s Current Law:
  \[ i_1 + i_3 + i_5 = I_{dc} \] (one switch always closed)
  \[ i_2 + i_4 + i_6 = I_{dc} \]
- Apply Kirchhoff’s Voltage Law:
  \[ e_{an} + e_{bn} + e_{cn} = 0 \] (balanced 3 phase set)
- Since $X_c = 0$, only one switch in (1,3,5) can be closed with a switch in (2,4,6)
Basic Six-Pulse Converter (cont.)

• Allowable combinations:
  1 with (2 or 6) (4 shorts dc bus)
  3 with (2 or 4)
  5 with (4 or 6)
  2 with (1 or 5)
  4 with (1 or 3)
  6 with (3 or 5)

• Need to determine a switching sequence

• Start from assumption of positive phase sequence

• Typical current waveforms:
\[
\begin{align*}
  i_a & \quad | \\
  i_b & \quad | \\
  i_c & \quad |
\end{align*}
\]

Basic Six-Pulse Converter (cont.)

• Possible sequences:
  Top three switches: 1-3-5-1 or 1-5-3-1
  Bottom three switches: 4-6-2-4 or 4-2-6-4

• Assume: \( V_{dc} = V_{dc}^+ - V_{dc}^- \)

<table>
<thead>
<tr>
<th>Switch #</th>
<th>( V_{dc}^+ )</th>
<th>Switch #</th>
<th>( V_{dc}^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( e_{an}(t) )</td>
<td>4</td>
<td>( e_{an}(t) )</td>
</tr>
<tr>
<td>3</td>
<td>( e_{bn}(t) )</td>
<td>6</td>
<td>( e_{bn}(t) )</td>
</tr>
<tr>
<td>5</td>
<td>( e_{cn}(t) )</td>
<td>2</td>
<td>( e_{cn}(t) )</td>
</tr>
</tbody>
</table>
Basic Six-Pulse Converter (cont.)

- Positive sequence (α = 0, 1-3-5-1 and 4-6-2-4)

- Negative sequence (α = 0, 1-5-3-1 and 4-2-6-4)

- Phase currents:

  \[
  I_{dc} = \begin{array}{c|c|c|c}
  5 & 1 & 3 & 5 \\
  \hline
  6 & 2 & 4 & \end{array}
  \]

- Look at the line voltages:

  \[
  V_{dc} = V_{dc}^+ - V_{dc}^-
  \]

  \[
  \begin{align*}
  e_{ab} &= e_{an} - e_{bn} = V_{dc} \\
  e_{ac} &= e_{cn} - e_{an} \\
  e_{bc} &= e_{bn} - e_{cn} \\
  e_{ba} &= e_{bn} - e_{an} \\
  e_{ca} &= e_{cn} - e_{an} \\
  e_{cb} &= e_{cn} - e_{bn}
  \end{align*}
  \]

- If α = 0, then \( V_{dc} = \frac{3\sqrt{2}}{\pi} |V_{LL}| = 1.35 |V_{LL}| \) We define this as \( V_{do} \)
Controlled Firing of Thyristors

• Now add a firing delay (\(\alpha\)) for the thyristors. Same delay for all 6 switches

\[
V_{dc} = \frac{3}{\pi} \int_{-\frac{\pi}{6} + \alpha}^{\frac{\pi}{6} + \alpha} \sqrt{2} |V_{LL}| \cos(\theta) d\theta = \frac{3\sqrt{2}}{\pi} |V_{LL}| \sin(\theta) \bigg|_{-\frac{\pi}{6} + \alpha}^{\frac{\pi}{6} + \alpha}
\]

• Then \(V_{dc} = \frac{3\sqrt{2}}{\pi} |V_{LL}| \cos \alpha\)

• Define \(V_{do} = \frac{3\sqrt{2}}{\pi} |V_{LL}|\)

• Therefore \(V_{dc} = V_{do} \cos \alpha\)

• \(\alpha = 0 \rightarrow\) diode bridge \(V_{dc} = V_{do}\)
  \(0 \leq \alpha < 90 \rightarrow\) rectifier \(V_{dc} > 0\)
  \(\alpha = 90 \rightarrow P = 0 V_{dc} = 0\)
  \(90 < \alpha \leq 180 \rightarrow\) inverter \(V_{dc} < 0\)

• Current does not reverse
Commutation Overlap

- Now add source inductance \((L_c \neq 0)\)

\[
\begin{align*}
L_s & \quad + \\
V_{dc} & \quad - \\
\end{align*}
\]

Current Transfer Between Switches

- Current does not fall to zero immediately in ac side inductance
- Temporarily create line to line short
Current Transfer Between Switches (cont.)

- What happens if $\alpha$ gets to big (i.e. $\alpha \Rightarrow 180^\circ$)?

\[
\begin{array}{c|c|c}
1 & & 1 \\
0 & I_\text{dc} &
\end{array}
\]

This is called a commutation failure

- Thyristor 3 fails to turn on and thyristor 1 fails to turn off
- This is more common if $L_c$ is large, which is the case looking into a “weaker” ac system
- Normally corrects during next interval, although often have a second failure when thyristor 5 turns on, “double commutation failure”

Output Voltage During Commutation

- Switch 1 contribution: $V_{dcl}^+ = e_{an} - L_c \frac{di_1}{dt}$
- Switch 3 contribution: $V_{dcl}^+ = e_{bn} - L_c \frac{di_3}{dt}$
- During overlap we see the average between $V_{dcl}^+$ & $V_{dcl}^+$

So $V_{dc}^+ = \frac{V_{dcl}^+ + V_{dcl}^+}{2} = \frac{e_{an} + e_{bn}}{2} - L_c \frac{1}{2} \left( \frac{di_1}{dt} + \frac{di_3}{dt} \right)$

- $i_1 + i_3 = I_c$, so $\frac{di_1 + di_3}{dt} = 0$
- But since its a linear network: $\frac{di_1 + di_3}{dt} = \left( \frac{di_1}{dt} + \frac{di_3}{dt} \right) = 0$
- So: $V_{dc}^+ = V_{dc}^+ - e_{cn} = \frac{e_{an} + e_{bn}}{2} - e_{cn} = \frac{e_{ac} + e_{bc}}{2}$
Average DC Voltage with Overlap

- Recall: \( V_{do} = \frac{3\sqrt{2}}{\pi} |V_{LL}| = \frac{3\sqrt{6}}{\pi} |V_{\phi}| = \frac{3\sqrt{3}}{\pi} |E_m| \)
  where \( E_m \) is peak line to neutral voltage

- Then we find:
  \[
  V_{dc} = \frac{3}{\pi} \left[ \frac{3}{2} E_m \cos \theta d\theta + \frac{\sqrt{3}}{2} E_m \cos \left( \theta - \frac{\pi}{6} \right) d\theta \right]
  \]

- Leading to: \( V_{dc} = \frac{3\sqrt{3}}{2\pi} E_m [\cos \alpha + \cos (\alpha + \mu)] \)

- Or \( V_{dc} = \frac{V_{do}}{2} [\cos \alpha + \cos (\alpha + \mu)] \)

Average DC Current

- Start out with \( L_c = 0 \) and \( \alpha = 0 \) for now

- Firing delay simply adds a phase shift to the current (always lagging), and \( \cos \alpha = \cos \phi \)

- Fundamental Component
  \[
  i_{1pk} = \frac{2}{\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} i_a \cos(\theta) d\theta = \frac{2}{\pi} I_{dc} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos(\theta) d\theta = \frac{2\sqrt{3}}{\pi} I_{dc}
  \]

- \( |I_{1RMS}| = \frac{\sqrt{6}}{\pi} I_{dc} \)
**Average DC Current**

- Then $i_1(t) = 2\sqrt{3} I_{dc} \cos(\omega t - \alpha)$
- Also: $P = 3I_{RM}V_p \cos \phi = V_{dc}I_{dc}$
  
  
  So: $3I_{RM}V_p \cos \phi = \frac{3\sqrt{6}}{\pi} V_p \cos \alpha I_{dc}$

- So: $|I_{a1RMS}| = \frac{\sqrt{6}}{\pi} I_{dc}$ as expected

- During overlap: $I_{dc} = I_c = \frac{\sqrt{3}E_m}{2\omega X_c} = \frac{\epsilon_m}{2X_c}$

- $i_3(t) = I_c(\cos \alpha - \cos \omega t)$ with $\alpha \leq \omega t \leq \alpha + \mu$
  
  where $\omega t = \alpha + \mu$ at the end of the commutation interval

- So average current is: $I_{dc} = I_c(\cos \alpha - \cos(\alpha + \mu))$

- Also: $I_c = \frac{\sqrt{3}E_m}{2\omega L_c} = \sqrt{\frac{3}{2}} \frac{|V_p|}{X_c} = \frac{|V_{LL}|}{\sqrt{2}X_c}$

**Average DC Circuit Equations**

- We have the following equations:

  \[
  V_{dc} = \frac{V_{do}}{2}[\cos \alpha + \cos(\alpha + \mu)]
  \]

  \[
  I_{dc} = I_c(\cos \alpha - \cos(\alpha + \mu))
  \]

  \[
  V_{do} = \frac{3\sqrt{2}}{\pi} |V_{LL}|
  \]

  \[
  I_c = \frac{|V_{LL}|}{\sqrt{2}X_c} = \frac{\pi V_{do}}{6X_c}
  \]

- Substitute for the $\cos(\alpha + \mu)$ in the $V_{dc}$ equation

- Then $V_{dc} = V_{do} \cos \alpha - \frac{V_{dc}}{2I_c} I_{dc}$

- Where $\frac{V_{dc}}{2I_c} = \frac{V_{do}}{2\pi X_c} = \frac{3}{\pi} X_c = R_c$ (called the commutating “resistance”)

- So $V_{dc} = V_{do} \cos \alpha - I_{dc}R_c$
Average DC circuit

- $R_c$ represents a current dependent voltage drop due to overlap
- $R_c$ does not represent any energy dissipation!
- So using $V_{dc} = V_{do} \cos \alpha - I_{dc} R_c$ we get:

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  \[ V_{dc} = V_{do}\cos\alpha - I_{dc} R_c \]
```

Inverter Operation

- $\alpha + \mu + \gamma = \pi$ Covers positive half cycle of voltage
- $\gamma$ is defined as the extinction angle
- $\gamma_o$ is minimum extinction angle for proper turn-off
  Typical values: $15^\circ \rightarrow 20^\circ$
- So $\alpha + \mu \leq 180^\circ - \gamma_o$ gives limits for control settings
- Replace $\alpha$ with $180^\circ - \mu - \gamma$ in averaged equations
  * note: $\cos(180^\circ - \theta) = -\cos(\theta)$
Inverter Operation (cont.)

• Generate equations in terms on $\gamma$ instead of $\alpha$

\[
V_{dc} = \frac{V_{do}}{2} [\cos \alpha + \cos(\alpha + \mu)]
\]
\[
V_{dc} = \frac{V_{do}}{2} [\cos(180^\circ - \mu - \gamma) + \cos(180^\circ - \gamma)]
\]
\[
V_{dc} = \frac{-V_{do}}{2} [\cos(\gamma + \mu) + \cos(\gamma)]
\]
\[
I_{dc} = I_c(\cos \alpha - \cos(\alpha + \mu))
\]
\[
I_{dc} = I_c(\cos \gamma - \cos(\gamma + \mu))
\]

• Sign reversal in voltage equation expected for inverter

Effect of Overlap on Power Transfer

• $P_{ac} = 3I_{1RMS}V_p \cos \phi$

• $P_{dc} = I_{dc} \cdot V_{do} \left[ \frac{\cos \alpha + \cos(\alpha + \mu)}{2} \right] = I_{dc} \frac{3\sqrt{6}V_p}{\pi} \left[ \frac{\cos \alpha + \cos(\alpha + \mu)}{2} \right]$

• $I_{1RMS} = I_{dc} \frac{\sqrt{6}}{\pi}$

• Then $\cos \phi = \frac{\cos \alpha + \cos(\alpha + \mu)}{2} = \frac{V_{dc}}{V_{do}}$

Transformer Loading

DC System

\[ X_c = \omega L_c \text{ where } X' \rightarrow 12 - 20\% \]

\[ Z_B = \frac{V_B}{I_B} \]

\[ X_c = Z_B X' = \frac{V_B}{I_B} X' = \frac{3V_B^2}{3V_B I_B} X' \text{, this is } \frac{V_{L_B}^2}{MVA_3}\phi_B \]

\[ X_c = \frac{V_{BL}^2}{MVA_B3\phi} X' \]

\[ I_B = I_{1\text{RMS}} = \frac{\sqrt{6}}{\pi} I_{dcB} \]

\[ \text{So } MVA_{B3\phi} = 3V_{B\phi} I_B = 3V_{B\phi} \left( \frac{\sqrt{6}}{\pi} I_{dcB} \right) = V_{do}^R I_{dcB} \]

Transformer Loading (cont.)

\[ I_{1\text{RMS}} = \sqrt{\frac{1}{\pi} \int_0^{2\pi} I_{dc}^2 d\theta} = I_{dc} \sqrt{\frac{2}{3}} \]

\[ \text{So } MVA_{B3\phi} = 3I_{1\text{RMS}} V_{\phi} = \frac{\pi}{3} V_{do}^R I_{dcB} \]

\[ \text{Then } X_c = \frac{V_{BL}^2}{MVA_B3\phi} X' \text{ with } V_{do} = \frac{3\sqrt{2}}{\pi} V_{BL} \]

\[ \text{Then } Z_B = \frac{V_{BL}}{\theta_{dcB}} \]

\[ \text{Then from } V_{dc} = V_{do} \cos \alpha - I_{dc} \frac{3}{\pi} X_c \text{ we get} \]

\[ V_{dc} = V_{do} \cos \alpha - I_{dc} \frac{3}{\pi} (Z_B X') \]

\[ V_{dc} = V_{do} \cos \alpha - V_{do} X' \frac{I_{dc}}{2 I_{dcB}} \]

\[ \text{Leading to a “per unit” expression: } \frac{V_{dc}}{V_{do}} = \cos \alpha - \frac{X'}{\frac{2}{I_{dc}}} \]