ECE 529
Utility Applications of Power Electronics
Session 37
$V_{dc} = V_{pospole} - V_{negpole}$

Diagram showing waveforms for $v_a(t)$, $v_b(t)$, $v_c(t)$, $v_{ca}(t)$, $v_{bc}(t)$, and $v_{ba}(t)$ with annotations indicating phase shifts and voltage relationships.
Commutation Overlap

- Now add source inductance ($L_c \neq 0$)

\[ l_1 + l_3 + l_5 = I_{dc} \]

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Current Transfer Between Switches

- Current does not fall to zero immediately in ac side inductance
- Temporarily create line to line short

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HVDC Transmission Systems
Current Transfer Between Switches (cont.)

- What happens if $\alpha$ gets to big (i.e. $\alpha \rightarrow 180^\circ$)?
  - **insufficient time for commutation**
  - This is called a commutation failure

- Thyristor 3 fails to turn on and thyristor 1 fails to turn off
- This is more common if $L_c$ is large, which is the case looking into a “weaker” ac system
- Normally corrects during next interval, although often have a second failure when thyristor 5 turns on, “double commutation failure”

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Output Voltage During Commutation

- Switch 1 contribution: $V_{dc1}^+ = e_{an} - L_c \frac{di_1}{dt}$
- Switch 3 contribution: $V_{dc3}^+ = e_{bn} - L_c \frac{di_3}{dt}$
- During overlap we see the average between $V_{dc1}^+$ & $V_{dc3}^+$

So $V_{dc}^+ = \frac{V_{dc1}^+ + V_{dc3}^+}{2} = \frac{e_{an} + e_{bn}}{2} - L_c \frac{di_1 + di_3}{2 dt}$

- $i_1 + i_3 = I_{dc}$, so $\frac{di_1 + di_3}{dt} = 0$
- But since its a linear network: $\frac{di_1 + di_3}{dt} = \left( \frac{di_1}{dt} + \frac{di_3}{dt} \right) = 0$

- So: $V_{dc} = V_{dc}^+ - e_{cn} = \frac{e_{an} + e_{bn}}{2} - e_{cn} = \frac{e_{ac} + e_{bc}}{2}$
Average DC Voltage with Overlap

- Recall: \( V_{do} = \frac{3\sqrt{2}}{\pi} |V_{LL}| = \frac{3\sqrt{2}}{\pi} |V_\phi| = \frac{3\sqrt{3}}{\pi} |E_m| \)
  where \( E_m \) is peak line to neutral voltage

- Then we find:
  \[
  V_{dc} = \frac{3}{\pi} \left[ \int_{\alpha}^{\alpha+\mu} \frac{3}{2} E_m \cos \theta d\theta + \int_{\alpha+\mu}^{\alpha+\mu+\frac{\pi}{6}} \sqrt{3} |E_m| \cos (\theta - \frac{\pi}{6}) d\theta \right]
  \]

- Leading to: \( V_{dc} = \frac{3\sqrt{3}}{2\pi} E_m [\cos \alpha + \cos (\alpha + \mu)] \)

- Or \( V_{dc} = \frac{V_{do}}{2}[\cos \alpha + \cos (\alpha + \mu)] \rightarrow \text{overlap reduces} \) <\( V_{dc} \)>

\( \mu > 0 \)

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Average DC Current

- Start out with \( L_c = 0 \) and \( \alpha = 0 \) for now

- Firing delay simply adds a phase shift to the current (always lagging), and \( \cos \alpha = \cos \phi \)

- Fundamental Component
  \[
  i_{1pk} = \frac{2}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} i_a \cos(\theta) d\theta = \frac{2}{\pi} I_{dc} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\theta) d\theta = \frac{2\sqrt{3}}{\pi} I_{dc}
  \]

- \( |I_{1RMS}| = \frac{\sqrt{6}}{\pi} I_{dc} \)
### Average DC Current

- Then \( i_1(t) = 2\frac{\sqrt{2}}{\pi} I_{dc} \cos(\omega t - \alpha) \)

- Also: \( P = 3I_{1RMS}V_p\cos\phi = V_{dc}I_{dc} \)
  \[ \text{So: } 3I_{1RMS}V_p\cos\phi = \frac{3\sqrt{6}}{\pi} V_p\cos\alpha I_{dc} \]
  \[ \text{So: } |I_{1RMS}| = \frac{\sqrt{6}}{\pi} I_{dc} \text{ as expected} \]

- During overlap: \( I_{dc} = I_c = \frac{\sqrt{3E_m}}{2\omega L_c} = \frac{E_m}{2X_c} \)

- \( i_3(t) = I_c(\cos\alpha - \cos\omega t) \) with \( \alpha \leq \omega t \leq \alpha + \mu \)
  where \( \omega t = \alpha + \mu \) at the end of the commutation interval

- So average current is: \( I_{dc} = I_c(\cos\alpha - \cos(\alpha + \mu)) \)

- Also: \( I_c = \frac{\sqrt{3E_m}}{2\omega L_c} = \sqrt{\frac{3}{2}} \frac{|V_p|}{X_c} = \frac{|V_{LL}|}{\sqrt{2}X_c} \)

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### Average DC Circuit Equations

- We have the following equations:

  \[ V_{dc} = \frac{V_{do}}{2}[\cos\alpha + \cos(\alpha + \mu)] \]

  \[ I_{dc} = I_c(\cos\alpha - \cos(\alpha + \mu)) \]

  \[ V_{do} = \frac{3\sqrt{2}}{\pi} |V_{LL}| \]

  \[ I_c = \frac{|V_{LL}|}{\sqrt{2}X_c} = \frac{\pi V_{do}}{6X_c} \]

- Substitute for the \( \cos(\alpha + \mu) \) in the \( V_{dc} \) equation

- Then \( V_{dc} = V_{do}\cos\alpha - \frac{V_{do}I_{dc}}{2L_c} \)

- Where \( \frac{V_{do}}{2L_c} = \frac{|V_{LL}|}{\pi X_c} = R_c \) (called the commutating “resistance”)

- So \( V_{dc} = V_{do}\cos\alpha - I_{dc}R_c \)

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*HVDC Transmission Systems*
**Average DC circuit**

- $R_c$ represents a current dependent voltage drop due to overlap
- $R_c$ does not represent any energy dissipation!
- So using $V_{dc} = V_{do} \cos \alpha - I_{dc} R_c$ we get:

![Diagram of a DC circuit with symbols](image)

**Inverter Operation**

- $\alpha + \mu + \gamma = \pi$ Covers positive half cycle of voltage
- $\gamma$ is defined as the extinction angle
- $\gamma^*$ is minimum extinction angle for proper turn-off
  Typical values: $15^\circ \rightarrow 20^\circ$
- So $\alpha + \mu \leq 180^\circ - \gamma^*$ gives limits for control settings
- Replace $\alpha$ with $180^\circ - \mu - \gamma$ in averaged equations
  *note: $\cos(180^\circ - \theta) = -\cos(\theta)$