ECE 529
Utility Applications of Power Electronics
Session 38
Average DC Current

• Then \( i_1(t) = \frac{2\sqrt{2}}{\pi} I_{dc}\cos(\omega t - \alpha) \)

• Also: \( P = 3I_{RMS}V_p\cos\phi = V_{dc}I_{dc} \)
  So: \( 3I_{RMS}V_p\cos\phi = \frac{3\sqrt{2}}{\pi} V_p\cos\alpha I_{dc} \)

• So: \( |V_{a1RMS}| = \frac{\sqrt{2}}{\pi} I_{dc} \) as expected

• During overlap: \( I_{dc} = I_c = \frac{\sqrt{3}E_{m}}{2oL} = \frac{6I_{dc}}{2X_c} \)

  \( i_3(t) = I_c(\cos\alpha - \cos\omega t) \) with \( \alpha \leq \omega t \leq \alpha + \mu \)
  where \( \omega t = \alpha + \mu \) at the end of the commutation interval

• So average current is: \( I_{dc} = I_c(\cos\alpha - \cos(\alpha + \mu)) \)

• Also: \( I_c = \frac{\sqrt{3}E_{m}}{2oL} = \sqrt{\frac{3}{2} \frac{|V_p|}{X_c}} = \frac{V_{LL}}{\sqrt{2X_c}} \)

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Average DC Circuit Equations

• We have the following equations:
  \[ V_{dc} = \frac{V_{do}}{2} [\cos\alpha + \cos(\alpha + \mu)] \]
  \[ I_{dc} = I_c(\cos\alpha - \cos(\alpha + \mu)) \]
  \[ V_{do} = 3\sqrt{2} |V_{LL}| \]
  \[ I_c = \frac{|V_{LL}|}{\sqrt{2X_c}} = \frac{\pi V_{do}}{6X_c} \]

• Substitute for the \( \cos(\alpha + \mu) \) in the \( V_{dc} \) equation

• Then \( V_{dc} = V_{do}\cos\alpha - \frac{V_{do}I_{dc}}{2\sqrt{2}L} \)

• Where \( \frac{V_{dc}}{2\sqrt{2}L} = \frac{\pi V_{do}}{6\sqrt{2}X_c} = R_c \) (called the commutating "resistance")

• So \( V_{dc} = V_{do}\cos\alpha - I_{dc}R_c \)
Average DC circuit

- $R_c$ represents a current dependent voltage drop due to overlap
- $R_c$ does not represent any energy dissipation!
- So using $V_{dc} = V_{do} \cos \alpha - I_{dc} R_c$ we get:

Inverter Operation

- $\alpha + \mu + \gamma = \pi$ Covers positive half cycle of voltage
- $\gamma$ is defined as the extinction angle
- $\gamma_0$ is minimum extinction angle for proper turn-off
  Typical values: $15^\circ \rightarrow 20^\circ$
- So $\alpha + \mu \leq 180^\circ - \gamma$ gives limits for control settings
- Replace $\alpha$ with $180^\circ - \mu - \gamma$ in averaged equations
  * note: $\cos(180^\circ - \theta) = -\cos(\theta)$

\[
\alpha + \mu + \gamma = 180 \\
\alpha = 180^\circ - \mu - \gamma
\]
\[ V_{dc \text{ave}} \text{ (pole to pole)} \]

\[ V_{dc \text{psp p}ole} - V_{dc \text{ne}g p}ole \]

\[ V_{d, \text{no \, cross}} = -V_{d, \text{off}} \cos \alpha \]
Inverter Operation (cont.)

- Generate equations in terms on $\gamma$ instead of $\alpha$

\[
V_{dc} = \frac{V_{do}}{2}[\cos\alpha + \cos(\alpha + \mu)] \\
V_{dc} = \frac{V_{do}}{2}[\cos(180^\circ - \mu - \gamma) + \cos(180^\circ - \gamma)] \\
V_{dc} = \left(-\frac{V_{do}}{2}\right)[\cos(\gamma + \mu) + \cos(\gamma)] \\
I_{dc} = I_c(\cos\alpha - \cos(\alpha + \mu)) = I_c\left(\cos(180^\circ - \mu - \gamma) - \cos(180^\circ - \gamma)\right) \\
I_{dc} = I_c(\cos\gamma - \cos(\gamma + \mu))
\]

- Sign reversal in voltage equation expected for inverter

Effect of Overlap on Power Transfer

- \( P_{ac} = 3I_{RMS}V_p\cos\phi \)
- \( P_{dc} = I_{dc}V_{do}\left[\frac{\cos\alpha + \cos(\alpha + \mu)}{2}\right] = \left(\frac{3\sqrt{6}V_p}{\pi}\right)\left[\frac{\cos\alpha + \cos(\alpha + \mu)}{2}\right] \)
- \( I_{RMS} = \frac{I_{dc}\sqrt{6}}{\pi} \)
- Then \( \cos\phi = \frac{\cos\alpha + \cos(\alpha + \mu)}{2} \left(\frac{V_{dc}}{V_{do}}\right) \)
Transformer Loading

\[ X_c = \omega L_c \] where \( X' \rightarrow 12-20\% \)

\[ Z_B = \frac{V_B}{I_B} \]

\[ X_c = Z_B X' = \frac{V_B}{I_B} X' = \frac{3V_B^2}{3V_B I_B} X', \text{ this is } \frac{V_B^2}{MVA_{3\phi}} \]

\[ X_c = \frac{V_B^2}{MVA_{3\phi}} X' \]

\[ I_B = I_{RMS} = \frac{\sqrt{6}}{\pi} I_{dcB} \]

\[ \text{So } MVA_{B3\phi} = 3V_B I_B = 3V_B \left( \frac{\sqrt{6} I_{dcB}}{\pi} \right) = V_{do}^R I_{dcB} \]

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Transformer Loading (cont.)

- Need to use true RMS (transformers sees all harmonic components)

\[ I_{RMS} = \sqrt{\frac{1}{\pi} \int_0^{2\pi} I_{dcB}^2 d\theta} = I_{dcB} \sqrt{\frac{2}{3}} \]

- So \( MVA_{B3\phi} = 3 I_{RMS} V_{\phi} = \frac{\pi}{3} V_{do}^R I_{dcB} \)

- Then \( X_c = \frac{V_B^2}{MVA_{B3\phi}} X' \) with \( V_{do} = \frac{3\sqrt{2}}{\pi} V_B L \)

- Then \( Z_B = \frac{\pi V_{do}}{6 I_{dcB}} \)

- Then from \( V_{dc} = V_{do} \cos \alpha - I_{dcB} \frac{3}{\pi} X_c \) we get

\[ V_{dc} = V_{do} \cos \alpha - I_{dcB} \frac{3}{\pi} (Z_B X') \]

\[ V_{dc} = V_{do} \cos \alpha - V_{do} \frac{X'}{2 I_{dcB}} \]

- Leading to a “per unit” expression:

\[ V_{do} = \frac{V_{dc}}{V_{do}^R} = \frac{X'}{2 I_{dcB}} \]