Decomposition of nucleon spin

$$\frac{1}{2} = \underbrace{\frac{1}{2}\Sigma}_{0.135} + L_q + \Delta g + L_g$$

$$\left.\begin{array}{ll} \frac{1}{2}\Sigma & \text{- quark spin} \\ L_q & \text{- orbital momentum} \end{array}\right\} J_q \text{ - total quark angular momentum}$$

$$\left.\begin{array}{ll} \Delta g & \text{- gluon spin} \\ L_g & \text{- orbital momentum} \end{array}\right\} J_g \text{ - total gluon angular momentum}$$

Experiment:

$\frac{1}{2}\Sigma \simeq 0.135$ measured

$\Delta g = \int_0^1 dx\, \Delta G(x)$ - will be measured soon

$J_q = \frac{1}{2}\Sigma + L_q$ - can be measured in DVCS (deeply virtual compton scattering)

Theory: speculations + QCD sum rules

$\Delta g \simeq 0.5$ (Brodsky, Burkardt, Schmidt)

$\Delta g \simeq 2$ (Mankiewicz et al, QCD sum rules)

$J_q \simeq J_g \simeq \frac{1}{4}$ (Balitsky & Ji, QCD sum rules)

No reliable lattice results yet.

Sum rule for nucleon spin

$\hat{J}_3 |p, \uparrow\rangle = \frac{1}{2} |p, \uparrow\rangle$ — nucleon at rest ($p = m,0,0,0$) with spin up OZ

$\hat{J}_3 = \int d^2x_\perp dx_- \; \hat{M}_{+12}(\vec{x})$ — lightcone quantization, $A_+ = 0$ gauge

$\hat{M}_{+12}$: QCD tensor of angular momentum

$\hat{M}_{+12} \equiv x_1 \hat{\Theta}_{+2} - x_2 \hat{\Theta}_{+1}$

$$\hat{\Theta}_{\mu\nu} = \underbrace{\frac{i}{4}\bar{\psi}(\overleftrightarrow{\nabla}_\mu \gamma_\nu + \overleftrightarrow{\nabla}_\nu \gamma_\mu)\psi}_{\text{quark part } \Theta^q_{\mu\nu}} \underbrace{- \frac{1}{4} G^a_{\mu\alpha} G^a_{\nu\alpha} + \frac{\delta_{\mu\nu}}{4} G^a_{\alpha\beta} G^a_{\alpha\beta}}_{\text{gluon part } \Theta^g_{\mu\nu}}$$

QCD energy-momentum tensor

Similarly

$\hat{M}^q_{\mu\nu\lambda} \equiv x_\nu \hat{\Theta}^q_{\mu\lambda} - x_\lambda \hat{\Theta}^q_{\mu\nu}$ — quark part of QCD angular mom. tensor

$\hat{M}^g_{\mu\nu\lambda} \equiv x_\nu \hat{\Theta}^g_{\mu\lambda} - x_\lambda \hat{\Theta}^g_{\mu\nu}$ — gluon part

$\nabla_\mu \equiv \partial_\mu - igA_\mu$

$$\hat{J}^q_3 \equiv \int d^2x_\perp dx_- \; \hat{M}^q_{+12} =$$

$$= \underbrace{\frac{1}{2}\int d^2x_\perp dx_- \; \bar{\psi}\gamma_+\gamma_5\psi}_{\frac{1}{2}\hat{\Sigma} \text{ - quark spin}} + i\underbrace{\int d^2x_\perp dx_- \; \bar{\psi}\gamma_+(x_1\nabla_2 - x_2\nabla_1)\psi}_{\hat{L}_q \text{ - quark orbital mom}}$$

$$\hat{J}^g_3 \equiv \int d^2x_\perp dx_- \; \hat{M}^g_{+12} = ?$$

In a free theory:

$K_+ \equiv A_1 \partial_+ A_2 - A_2 \partial_+ A_1$

$$\hat{J}^g_3 = \underbrace{\int d^2x_\perp dx_- \; K_+}_{\text{spin}} + \underbrace{\int d^2x_\perp dx_- \; G^a_{+i}(x_1\partial_2 - x_2\partial_1)A^a_i}_{\text{orbital}}$$

$\Rightarrow$ sum rule

$$\langle p,\uparrow | J_3 | p,\uparrow\rangle = \lim_{p'\to p} \langle p',\uparrow | J_3 | p,\uparrow\rangle = \frac{1}{2} \lim_{p'\to p} 2p_+ \delta^{(3)}(p'-p)$$

$$\overset{\displaystyle \frac{1}{2}\hat{\Sigma} + \hat{L}_q + \hat{J}_g}{\|} \quad \Rightarrow$$

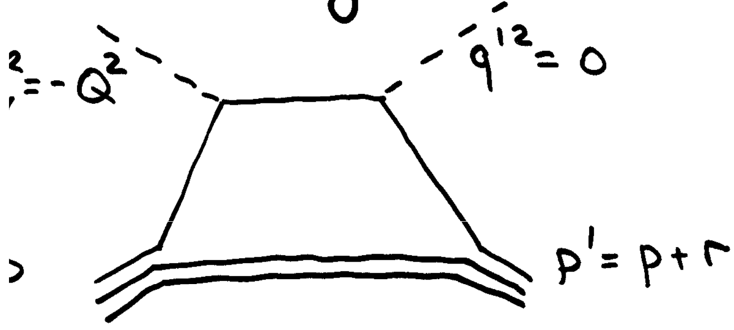
$$\Rightarrow \frac{1}{2} = \frac{1}{2}\Sigma + L_q + J_g = \frac{1}{2} \frac{\langle p,\uparrow | \bar{\psi}\gamma_+\gamma_5\psi | p,\uparrow\rangle}{2p_+} +$$

$$+ \lim_{p'\to p} \frac{1}{(2\pi)^3 2p_+ \delta^{(3)}(p'-p)} \langle p',\uparrow | \int d^2x_\perp dx_- \bar{\psi}\gamma_+(x_1\nabla_2 - x_2\nabla_1)\psi | p,\uparrow\rangle$$

$$+ \lim_{p'\to p} \frac{1}{(2\pi)^3 2p_+ \delta^{(3)}(p'-p)} \langle p',\uparrow | \int d^2x_\perp dx_- (x_1\theta^g_{+2} - x_2\theta^g_{+1}) | p,\uparrow\rangle$$

Quark angular momentum and DVCS

(X. Ji)



Amplitude of DVCS is determined by nonforward parton distributions $n \uparrow\uparrow q'$, $n^2=0$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p'|\bar\psi(-\tfrac{\lambda n}{2})\gamma_\mu \psi(\tfrac{\lambda n}{2})|p\rangle = H(x,\xi,r^2)\bar u(p')\gamma_\mu u(p) + E(x,\xi,r^2)\bar u(p')\frac{\sigma_{\mu\nu}r_\nu}{2m}u(p)$$

$$\xi = \frac{-\bar P\cdot q + \sqrt{(\bar P\cdot q)^2 + \bar M^2 Q^2}}{\bar M^2}$$

$\bar M^2 = M^2 - r^2/4$; $\bar P = \frac{p+p'}{2}$

At $\xi = r^2 = 0$ $H(x,0,0) = q(x)$ – usual quark parton distribution

As in the case of usual DIS, second moment of the nonforward parton distribution is related to matrix element of $\theta^q_{\mu\nu}$

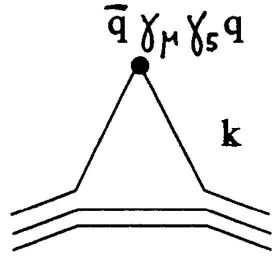
$$\langle p'|\theta^q_{\mu\nu}|p\rangle = \bar u(p')\{A^q(r^2)(\gamma_\mu \bar P_\nu + \gamma_\nu \bar P_\mu) + B^q(r^2)\frac{i}{2m}(\bar P_\mu \sigma_{\nu\alpha} r_\alpha + \mu \leftrightarrow \nu) + C^q(r^2)(r_\mu r_\nu - r^2\delta_{\mu\nu}) + D^q(r^2)\delta_{\mu\nu}\}u(p)$$

$$\int_{-1}^{1} dx\, x\{H(x,\xi,r^2) + E(x,\xi,r^2)\} = A^q(r^2) + B^q(r^2)$$

$$\langle p+r|M^q_{+12}|p\rangle = J_q (2\pi)^3 2p_+ \delta^3(r) \Rightarrow J_q = \tfrac{1}{2}(A^q(0) + B^q(0))$$

$r^2=0$ – nonphysical region for DVCS
$\Rightarrow$ extrapolation required

ANOMALY AND SPIN CRISIS

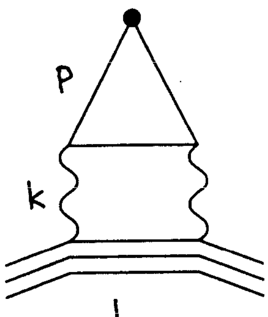


Interpretation of the matrix element as an integral over the parton density

$$\langle N|\bar{q}\gamma_\mu\gamma_5 q|N\rangle = 2S_\mu \int_0^1 dx(\ \Delta q(x) + \ \Delta\bar{q}(x))$$

assumes that all the tranverse momenta in the loop are $\sim M_N$.

But there is a contribution to $\langle N|\bar{q}\gamma_\mu\gamma_5 q|N\rangle$ coming from the region of large momenta ($p \sim$ UV cutoff $\sim$ Q)



$$\Longrightarrow \quad \frac{\alpha_s}{2\pi}\ \Delta g$$

gluons in the nucleon
$k_\perp^2 \sim \Lambda_{QCD}^2 \quad >> m_q^2$

$$\Delta\ g = \int_0^1 d\,x\,\Delta G(x)$$ -first moment of gluon spin distribution

If Δ g is large, everything may be OK still with old intuitive picture

$$\tilde{j}_\mu = j_\mu + \frac{\alpha_s}{2\pi} K_\mu$$ - conserved current

describes quark-partons and corresponds to loop momenta $k_\perp \sim M_N$

$$\Delta\tilde{q} = \langle N|\tilde{j}_\mu|N\rangle \qquad \Delta q = \Delta\tilde{q} - \frac{\alpha_s}{2\pi}\Delta\ g \qquad \Longrightarrow$$

It may be that $\Delta\tilde{s} \simeq 0$ and all Δ s $= -0.1$ comes from

$$\Delta\ s = \underbrace{\Delta\tilde{s}}_{\simeq 0} - \frac{\alpha_s}{2\pi}\Delta\ g \qquad \Longrightarrow \qquad \Delta g \sim \ \ 2 - 3$$

However , this seems unnatural: $\quad 1/2 = \underbrace{1/4}_{\Delta_q + L_q} + \ (\underbrace{3}_{\Delta g} - \underbrace{2.75}_{L_g})$

K_μ and gluon spin

$$x_\mu x_\nu \langle p,s| G^a_{\mu\alpha}\left(\tfrac{x}{2}\right) \tilde{G}^a_{\nu\alpha}\left(-\tfrac{x}{2}\right)|p,s\rangle = \qquad x^2=0$$

$$= -i(px)(sx)\int_0^1 du\; u\,\Delta G(u)\,\sin u(px)$$

↖ polarized gluon distribution

$$\Delta g = \int_0^1 du\,\Delta G(u)$$

$$\Rightarrow \langle p,s|\int_0^\infty d\lambda\; x_\mu G^a_{\xi\mu}(\lambda x)\, x_\nu \tilde{G}^a_{\nu\xi}(0)|p,s\rangle = (xs)\,\Delta g$$

$$K_\mu \equiv \frac{\alpha_s}{2\pi}\epsilon_{\mu\nu\rho\sigma} A^a_\nu\left(\partial_\rho A^a_\sigma + \frac{g}{3} f_{abc} A^b_\rho A^c_\sigma\right) \; - \text{ topological current}$$

In the light cone gauge $\quad n_\mu A_\mu = 0$

$$n_\mu K^{l.c.}_\mu = \int_0^\infty d\lambda\; n_\mu G^a_{\xi\mu}(\lambda n)\, n_\nu \tilde{G}^a_{\nu\xi}(0) \Rightarrow$$

$$\Rightarrow \langle p,s| n_\mu K^{l.c.}_\mu |p,s\rangle = -\frac{\alpha_s}{2\pi}(sn)\,\Delta g$$

Sum-rule estimation of Δg (Mankiewicz, Piller, Saafeld)

$$\int_0^1 du\; u^2\,\Delta G(u) \sim \langle N| G_{\mu\alpha} \mathcal{D}_\alpha \tilde{G}_{\nu\alpha}|N\rangle \leftarrow \text{calculated}$$

$$\int_0^1 du\; u^4\,\Delta G(u) \sim \langle N| G_{\mu\alpha} \mathcal{D}\mathcal{D}\mathcal{D} \tilde{G}_{\nu\alpha}|N\rangle \leftarrow \text{calculated}$$

$$\frac{\Delta G(x)}{G(x)} \xrightarrow[x\to 0]{} x \qquad \leftarrow \text{assumed}$$

$$\Rightarrow \Delta g(\mu\sim 1\,\text{GeV}) \simeq 2$$

Soon it will be measured!

From V.W. Hughes et al.
Amsterdam, 1996

Experiments Planned To Measure ΔG			
EXPERIMENT	**SLAC**	**COMPASS @CERN**	**RHIC**
Quantity Measured	$\mathcal{A}^{c\bar{c}}_{\vec{\gamma}\vec{N}} + \mathcal{A}^{J/\psi}_{\vec{\gamma}\vec{N}} + \mathcal{A}^{Bet-Heit}_{\vec{\gamma}\vec{N}} + \ldots$ 4 (e^- beam) energies	$\mathcal{A}^{\mu c\bar{c}}_{\vec{\mu}\vec{N}}$ up to 4 ν bins	$\mathcal{A}^{\gamma jet}_{\vec{p}\vec{p}}$ several x_G bins
Processes	$\vec{\gamma} + \vec{N} \rightarrow c\bar{c}$ $c \rightarrow \mu$ (BR= 8%) $\mu^+\mu^-$ & μ (high p_T)	$\vec{\mu} + \vec{N} \rightarrow \vec{\mu} + c\bar{c}$ $c \rightarrow D^0 \rightarrow K^-\pi^+$ (BR= 4%) Also $D^{*+} \rightarrow \pi^+ D^0$	$\vec{p} + \vec{p} \rightarrow \gamma$+jet
Kinematical range	Bremsstrahlung $\gamma's$, $Q^2 = 0$ $E^{min}_\gamma < E_\gamma < 48.5$ $0.10 < x_G < 0.25$	Quasi-real $\gamma's$ $Q^2 \approx 0$ $35 < \nu < 85$ $0.06 < x_G < 0.35$	$0.0 < x_G < 0.4$
Theoretical Basis & Uncertainties	LO available, NLO in progress For $\vec{\gamma} + \vec{N} \rightarrow c\bar{c}$ c quark mass uncertainty	LO available, NLO in progress For $\vec{\mu} + \vec{N} \rightarrow \vec{\mu} + c\bar{c}$ c quark mass uncertainty	For $qg \rightarrow \gamma(qjet)$ Background from $q\bar{q} \rightarrow \gamma(gjet)$; Should know Δq
Kinematical Constraints	Cuts on $p_T^{\mu\mu'}$, $M^{\mu\mu'}$ and p_T^{μ}	Events at D^0 mass	$5 < p_T < 30$
Experimental Difficulties	Disentangle $\mathcal{A}^{c\bar{c}}_{\gamma}$ from background asymmetry	Combinatorial background from K/π $B/S \approx 4$	Identify direct $\gamma's$; Contamination from $\pi^0 \rightarrow \gamma\gamma$
Statistical Error on $\mathcal{A}$	$\delta\mathcal{A}^{c\bar{c}}_{\gamma} = 0.01 - 0.02$	$\delta\mathcal{A}^{\mu c\bar{c}}_{\gamma} = 0.05$ for full data	$\delta\mathcal{A} = 0.002 - 0.04$
on $\Delta G/G$	$\delta(\Delta G/G) = 0.02 - 0.08$	$\delta < \Delta G/G >= 0.10$	$\delta(\Delta G/G) = 0.01 - 0.3$
Systematics	Contribution of backgrounds and randoms to $\mathcal{A}^{c\bar{c}}_{\gamma}$	Beam & target polarizations $\pm 4\%$	Beam polarization $\pm 6\%$ False asymmetries small
Status	pre-proposal stage	Approved by SPSLC	RHIC complete with Siberian snakes in 1999
Time scale	< Year 2000	$\geq$ Year 2000	Accelerator and detectors ready after year 2000
Remarks	Data taking : few months	Apparatus shared with hadron program	Apparatus shared with heavy ion program

Table 1.2: Gluon polarization experimental proposals.

Experiments Planned To Measure ΔG

EXPERIMENT	POLARIZED HERA Inclusive	POLARIZED HERA Exclusive (2-jets)	HERA-N
Quantity Measured	$g_1^p(x)$ wide x-Q^2 range	$\mathcal{A}_{\vec{e}\vec{p}}^{e(2\ jets)}$ several x_G bins	$\mathcal{A}_{\vec{p}\vec{N}}^{\gamma jet}$ & $\mathcal{A}_{\vec{p}\vec{N}}^{J/\psi jet}$ several x_G bins
Process	Polarized inclusive e,p DIS	$\vec{e}+\vec{p}\to 2$ jets Photon-Gluon-Fusion $(80-90\%)$	$\vec{p}+\vec{N}\to\gamma(J/\psi)+jet$ Internal $\vec{N}$ target
Kinematics Range	$1.8<Q^2<(1.8\times 10^4)$ $(5.5\times 10^{-5})<x<1$	$5<Q_G^2<100, \sqrt{s_{ij}}>10$ $0.002<x_G<0.2$	$0.1<x_G<0.4$
Theoretical Basis & Uncertainties	$\Delta G(x,Q^2)$ & $\int\Delta G$ from pQCD at NLO minimum uncertainties	LO calculations for $\mathcal{A}_{\vec{e}\vec{p}}^{e(2\ jets)}$ Lack of NLO calculations for polarized cross sections and for Monte Carlo	Onset of pQCD for $\gamma+(X)$; pQCD for $J/\psi+(X)$ Should know Δq
Kinematical Constraints	$y>0.01$ $\theta_{e'}>3^\circ$ $Q^2>1, E_{e'}>5$	$p_T>5, \mid\eta\mid<2.8$ $0.3<y<0.8$	$2\le p_T\le 8$ $-1.5\le\eta\le+1.5$
Experimental Problems	Polarization of 820 GeV protons in HERA and measurement of proton polarization	Gluon Compton 2-jet background	Identify direct γ's
Statistical Error on $\mathcal{A}$	for $\mathcal{L}=200$ $\delta\mathcal{A}=10^{-3}$ to 0.1	for $\mathcal{L}=200$ $\mathcal{A}=$ few%, $\delta\mathcal{A}<(0.2$ *to* $1\%)$	
on $\Delta G/G$	Relative error on $\int\Delta G$ 25(20)% with $\mathcal{L}=200(1000)$	$\delta(\Delta G/G)$ $0.10-0.50$	for $\mathcal{L}=250$ $\delta(\Delta G/G)<0.1$
Systematics	Measurement of P_e, P_p ($\pm 5\%$). False asymmetries small since can provide any sign of P_p for any bunch, and with a spin rotator can change P_p sign of all bunches.		
Status	Study of polarized protons at HERA; pre-proposal stage		
Time Scale	$\geq$ Year 2003 HERA operational with 27 GeV $\vec{e}$; H1 & ZEUS detectors operational		
Remarks Conclusions	Low x behaviour of $g_1^p(x,Q^2)$ of great interest	x_G is directly measured over a widekinematic range	Need (new) HERA-B type detector

Table 1.3: Gluon polarization experimental proposals (cont'd).

J_g (and J_q) from QCD sum rules

(I. Balitsky & X. Ji)

To calculate J_g consider

$$W^g_{\mu\nu\alpha} = \int dx\,dz\, e^{ipx} \langle 0|T\{\eta(x)\bar\eta(0)\, M^g_{\mu\nu\alpha}(z)\}|0\rangle$$

$$\eta(x) = \epsilon_{abc}(u^a C\gamma_\alpha u^b)\gamma_5\gamma_\alpha d^c$$

Useful trick:

$$W^g_{\mu\nu\alpha} = \int dx\,dz\, z^\mu z^\nu \langle 0|T\{\eta(x)\bar\eta(0)\, \mathcal{O}_\alpha(z)\}|0\rangle - (\nu\leftrightarrow\alpha)$$

$$\mathcal{O}_\alpha \equiv \partial_\mu \Theta^g_{\mu\alpha} = g\bar\psi G_{\alpha\beta}\gamma_\beta\psi$$

Sum rule:

$$W^{g\,(\mathrm{exp})}_{\mu\nu\alpha}(p) \overset{p^2\sim -1\,\mathrm{GeV}^2}{\simeq} W^{g\,(\mathrm{theor})}_{\mu\nu\alpha}$$

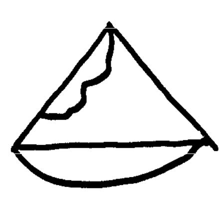
$$W^{g\,(\mathrm{exp})}_{\mu\nu\alpha}(p) = \left(\frac{\lambda_N^2 J_g}{(m_N^2-p^2)^2} + \frac{\mathrm{const}}{m_N^2-p^2} + \text{continuum}\right)\cdot 2i\,\gamma_\nu \underbrace{\not p}_{p_\mu}\ldots$$

$$\langle 0|\eta|N\rangle \sim \lambda_N$$

$$W^{g\,(\mathrm{theor})}_{\mu\nu\alpha}(p) = \text{pert. terms} + \text{local power corr.} + \text{bilocal power corrections}$$

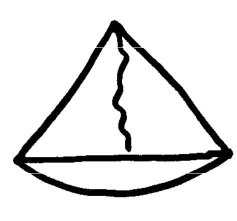
Pert. d-ms:

$\mu \equiv \mu_{UV}$



$$\frac{\alpha_s}{\pi^5} p^2 \left(\frac{1}{144}\ln^2\frac{\mu^2}{p^2} + \frac{1}{36}\ln\frac{\mu^2}{p^2}\right)$$

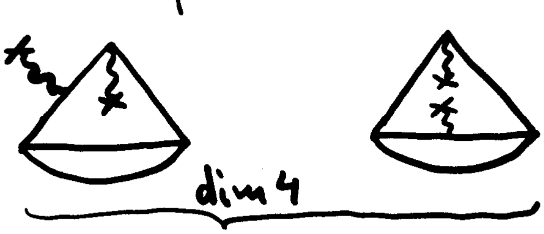
(less than 10% of final answer)

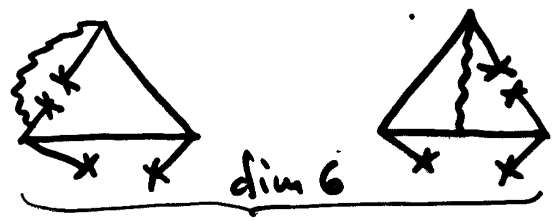


not calculated

(presumably small)

Local power corrections



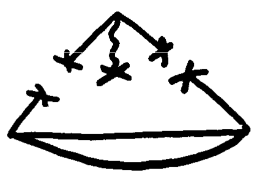


$$-\frac{\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2\rangle}{144\pi^2 p^2}\left(\ln\frac{p^2}{\sigma^2}+\frac{7}{6}\right) \qquad \frac{\alpha_s\langle\bar\psi\psi\rangle^2}{81\pi p^4}\left(62\ln\frac{\mu^2}{\sigma^2}-20\ln\frac{\mu^2}{p^2}\right)$$

$\sigma \equiv \sigma_{IR}$
IR cutoff.

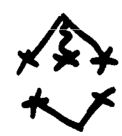
Bilocal power corrections

dim 4 :



$$\longrightarrow -\frac{1}{12\pi^2 p^2}\Pi_0(0,\mu^2)$$

$\Pi_0 \equiv$



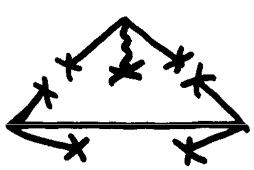
– two-point correlator at $q=0$

Estimate

$$\Pi_0(0,s_V) \simeq 1.1\cdot 10^{-3}\,\mathrm{GeV}^4 \left(+\langle\frac{\alpha_s}{\pi}G_{\mu\nu}^2\rangle\frac{1}{12}\ln\frac{\sigma^2}{s_V}\right)$$

m_R – mass scale in the exotic 1^{-+} channel
s_V – continuum threshold in that channel

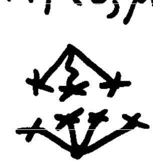
dim6



$$\longrightarrow \frac{4}{3p^4}\Pi_1(0,\mu^2)$$

← dominant contribution

$\Pi_1 \equiv$



Estimate

$$\Pi_1(0,s_V) \simeq \frac{m_0^2\langle\bar\psi\psi\rangle^2}{3m_R^2}\left(+\frac{31}{54}\frac{\alpha_s}{\pi}\langle\bar\psi\psi\rangle^2\ln\frac{\sigma^2}{s_V}\right)$$

$m_R \simeq 1.5$ GeV , $s_V \simeq 1.9$ GeV $\qquad m_0^2 = \frac{\langle\bar\psi\sigma G\psi\rangle}{\langle\bar\psi\psi\rangle} \simeq 0.65$

Sum rule:

$$\frac{J_g \lambda^2}{(m_N^2-p^2)^2} + \frac{\text{const}}{m_N^2-p^2} + \text{continuum} =$$

$$= \frac{\alpha_s}{\pi^5} p^2 \left(\frac{\ln^2 \mu^2/p^2}{144} + \frac{\ln \mu^2/p^2}{36} \right) + \frac{\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \rangle}{144\pi^2 p^2} \left(\ln \frac{\mu^2}{p^2} - \frac{7}{6} \right)$$

$$- \frac{1.1 \times 10^{-3}\,\text{GeV}^4}{12\pi^2 p^2} - \frac{20\alpha_s}{81\pi} \frac{\langle \bar\psi\psi \rangle^2}{p^4} \ln \frac{\mu^2}{p^2} + \boxed{\frac{4 m_0^2 \langle \bar\psi\psi \rangle^2}{9 m_R^2 p^4}}$$

dim 8 = 0 in the factorization approximation

Multiplication by $m_N^2 - p^2$ + Borel transformation
$\Rightarrow$ single-pole term drops

Answer

$$J_g(\mu = 1\,\text{GeV}) \simeq 0.35 \pm 0.15$$

If we keep only the dominant contribution we obtain

$$J_g(\mu = 1\,\text{GeV}) = \frac{8 e\, m_0^2 \langle \bar\psi\psi \rangle^2}{9 m_R^2 \lambda_N^2} = 0.25$$

We interprete this as $J_g \simeq J_q \simeq \frac{1}{4}$ with our accuracy.